UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS Part II

May–June 2020 **1 hour 15 minutes**

PHYSICS (7)
PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (7)

QUANTUM CONDENSED MATTER PHYSICS

Candidates offering this paper should attempt a total of ***four*** *questions:* ***three*** *questions from Section A and* ***one*** *question from Section B.*

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains ***four*** *sides, including this coversheet. You may use the formula handbook for values of constants and mathematical formulae, which you may quote without proof.*

You have 75 minutes (plus any pre-agreed individual adjustment) to answer this paper. Do not start to read the questions on the subsequent pages of this question paper until the start of the time period.

Please treat this as a closed-book exam and write your answers within the time period. Downloading and uploading times should not be included in the allocated exam time. If you wish to print out the paper, do so in advance. You can pause your work on the exam in case of an external distraction, or delay uploading your work in case of technical problems.

Section A and the chosen section B question should be uploaded as separate pdfs. Please name the files 1234X_Qi.pdf, where 1234X is your examination code and i is the number of the question/section (A or 4 or 5).

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Master coversheet	Mathematical Formulae handbook
	Approved calculator allowed

QUANTUM CONDENSED MATTER PHYSICS

SECTION A

*Attempt **all** questions in this Section. Answers should be concise and relevant formulae may be assumed without proof.*

A1 Consider a hole formed by removing an electron from a particular Bloch state in an otherwise filled band. What are the momentum, energy, velocity, effective mass and charge of the hole compared with those of an electron occupying this particular Bloch state in an otherwise empty band? [4]

A2 Calculate the cyclotron frequency for a hole in GaAs at the centre of the Brillouin zone for a magnetic field $B = 1$ T, assuming $\mathrm{d}^2E(k)/\mathrm{d}k^2 \approx 2.44 \times 10^{-38}\,\mathrm{m^4\,kg\,s^{-2}}$. [4]

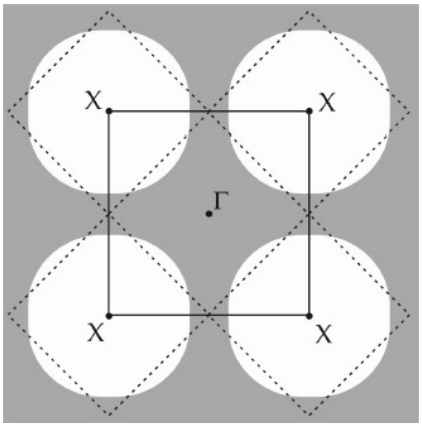
A3 A current is passed through an n-doped semiconductor for which the electron mobility is $0.2\,\mathrm{m^2\,V^{-1}\,s^{-1}}$. At what magnetic field, applied perpendicular to the current flow, does the Hall field reach 1% of the electric field along the current flow? [4]

SECTION B

Attempt one question from this section

B4 (a) Explain why a Fermi surface can normally only cross the Brillouin-zone boundary at right angles. Sketch the expected Fermi surface for a two-dimensional metal for different values of the conduction electron density, indicating clearly the electron surfaces, hole surfaces, and van Hove singularities. [6]

A copper-oxide high-temperature superconductor, P, can be modelled as a metal with conductivity in only two dimensions, with a single approximately cylindrical Fermi surface for holes, centred at the corners X of the first Brillouin zone. The Brillouin zone is shown by the solid square in the figure with its centre at Γ:



(b) The superconductor P has a square lattice with lattice constant $a = 0.386$ nm. In a magnetic field B, it exhibits quantum oscillations with the period $\Delta_{\mathrm{P}}(1/B) = 5.5 \times 10^{-5}\ \mathrm{T}^{-1}$. Using the Onsager relation for the extremal cross-sectional area A_k of the Fermi surface,

$$A_k = \frac{2\pi e}{\hbar} \frac{1}{\Delta(1/B)},$$

deduce the hole concentration per site in the superconductor. [3]

(c) Another type of copper-oxide superconductor, Q, with the same lattice constant as P, has a hole concentration $p = 1.1$ per site, and exhibits quantum oscillations with a period $\Delta_{\mathrm{Q}} = 1.85 \times 10^{-3}\ \mathrm{T}^{-1}$. Assuming Q has a single cylindrical Fermi surface, as in P, compare the expected periodicity of quantum oscillations in Q with the measured periodicity Δ_{Q}, and comment on your result. [2]

(d) It is suggested that there may be a doubling of the unit cell in Q, causing the Brillouin zone to fold back, giving the Brillouin-zone scheme indicated by the dashed lines in the figure above. Obtain the number of holes per site implied by this suggestion, and discuss the validity of the modified Brillouin zone as an explanation for the observed quantum- oscillation periodicity Δ_{Q}. [8]

(TURN OVER

B5 (a) What is meant by 'screening' of electrostatic fields inside a conductor? Explain how screening arises and why it is not possible for the conductor to perfectly screen an electric field at short ranges. [2]

(b) Outline the approximations made in the Thomas-Fermi theory of screening and explain why the chemical potential μ is expressed in this theory as

$$E_{\mathrm{F}}(\mathbf{r}) = \mu + eV(\mathbf{r})\,, \qquad [2]$$

where E_{F} is the Fermi energy calculated within the free-electron model for the local electron density $n(\mathbf{r})$, e is the electronic charge and $V(\mathbf{r})$ is the electrostatic potential.

(c) Certain three-dimensional materials have a linear dispersion relation for the conducting electrons: $E(\mathbf{k}) = \hbar v|\mathbf{k}|$, where v is a constant. Show that within the Thomas-Fermi approximation and for small changes in $n(\mathbf{r})$ from its value n_0 found when V is zero, the excess local charge density induced by the electrostatic potential V is given by

$$\rho_{\mathrm{ind}}(\mathbf{r}) = -e[n(\mathbf{r}) - n_0] \approx -\frac{3e^2 n_0}{E_{\mathrm{F0}}} V(\mathbf{r})\,,$$

where E_{F0} is the Fermi energy of free electrons with number density n_0 in this material. [6]

(d) Poisson's equation for the electrostatic potential associated with the charge distribution $\rho(\mathbf{r})$ in the presence of additional induced charges $\rho_{\mathrm{ind}}(\mathbf{r})$ is given by

$$\nabla^2 V(\mathbf{r}) = -\frac{1}{\varepsilon_0 \varepsilon}[\rho_{\mathrm{ind}}(\mathbf{r}) + \rho(\mathbf{r})]\,.$$

Show that, within the Thomas-Fermi approximation, a screened potential around a point charge of the form $V(r) \propto 1/r\ \exp(-r/\xi)$ satisfies Poisson's equation, and show that the Thomas-Fermi screening length ξ is given by

$$\xi = \sqrt{\frac{\varepsilon_0 \varepsilon E_{\mathrm{F0}}}{3e^2 n_0}}\,. \qquad [4]$$

(e) Evaluate ξ for a material that has a density of $2000\,\mathrm{kg\,m^{-3}}$, relative atomic mass of 65, one valence electron per atom, $\varepsilon = 20$ and $v = 80\,\mathrm{km\,s^{-1}}$. To what extent are the approximations that underlie the Thomas-Fermi theory valid for such a material? [5]