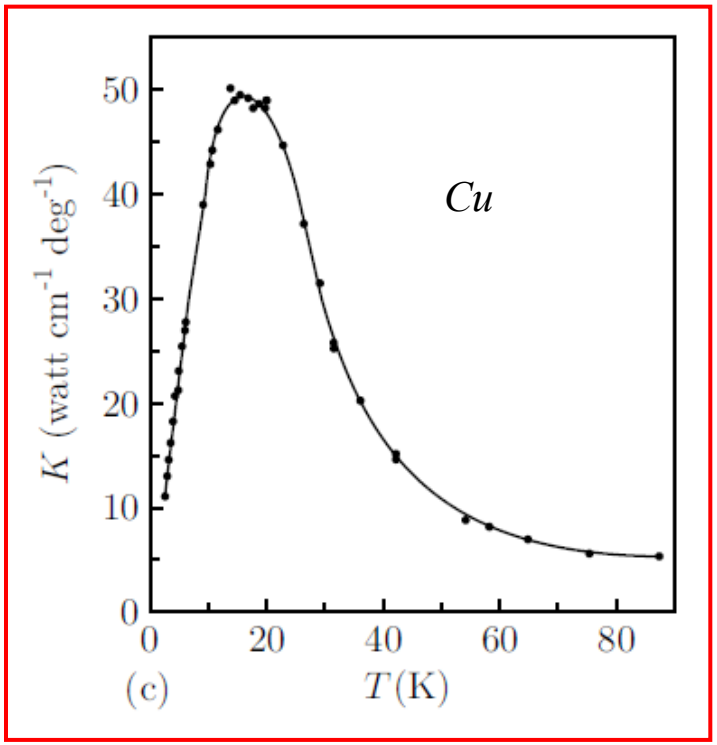
Quantum Condensed Matter Physics

Lecture 13



David Ritchie

Quantum Condensed Matter Physics

1. Classical and Semi-classical models for electrons in solids (3L)
2. Electrons and phonons in periodic solids (6L)
3. Experimental probes of band structure (4L)

Photon absorption; transition rates, experimental arrangement for absorption spectroscopy, direct and indirect semiconductors, excitons. Quantum oscillations; de Haas-Van Alphen effect in copper and strontium ruthenate. Photoemission; angle resolved photoemission spectroscopy (ARPES) in GaAs and strontium ruthenate. Tunnelling; scanning tunnelling microscopy. Cyclotron resonance. *Scattering in metals; Wiedemann-Franz law, theory of electrical and thermal transport, Matthiessen's rule, emission and absorption of phonons. Experiments demonstrating electron-phonon and electron–electron scattering at low temperatures.*

4. Semiconductors and semiconductor devices (5L)
5. Electronic instabilities (2L)
6. Fermi Liquids (2L)

Scattering in metals

- First we consider the effect of scattering in a metal with isotropic bands characterised by an effective mass m^* and a spherical Fermi surface
- Earlier in the course we discussed the relaxation time approximation finding an expression for electrical conductivity in terms of the electron number density, n, effective mass and scattering time for electrical conduction, τ_σ

$$\sigma = ne^2\tau_\sigma / m^*$$

- The thermal conductivity, κ is defined by $\mathbf{J}_q = -\kappa\nabla T$ where $\mathbf{J}_q$ is the flux of heat (energy per unit area per unit time)
- We also have from kinetic theory $\kappa = \frac{1}{3}\left\langle v^2 \right\rangle \tau_\kappa C_{el}$, τ_κ is the scattering time for thermal transport, C_{el} is the electronic specific heat given by:

$$C_{el} = \tfrac{1}{2} n\pi^2 k_B^2 T / E_F$$

- If we assume $\left\langle v^2 \right\rangle = v_F^2$ where v_F is the Fermi velocity then we obtain for the electronic thermal conductivity

$$\kappa = \tfrac{1}{6} n\pi^2 k_B^2 \frac{T}{E_F} v_F^2 \tau_\kappa$$

- Assuming $E_F = \frac{1}{2} m^* v_F^2$, taking the ratio of thermal to electrical conductivity and dividing by T we can define:

$$L = \frac{\kappa}{\sigma T} = \frac{\pi^2 k_B^2}{3e^2} \frac{\tau_\kappa}{\tau_\sigma}$$

Wiedemann-Franz law

- From the previous slide $L = \frac{\kappa}{\sigma T} = \frac{\pi^2 k_B^2}{3e^2}\frac{\tau_\kappa}{\tau_\sigma}$
- If we assume the scattering times are the same $\tau_\kappa = \tau_\sigma$ we have the *Wiedemann-Franz* law, which states that for metals the ratio of the thermal conductivity to the electrical conductivity is proportional to the temperature
- L_0 is the *Lorentz number* with value $L_0 = (\pi^2 k_B^2 / 3e^2) = 2.45 \times 10^{-8} W\Omega K^{-2}$
- Experimental results for T=0 and 100°C (see table) are in good agreement with this value for a range of different metals
- So by measuring the electrical conductivity we can estimate the thermal conductivity

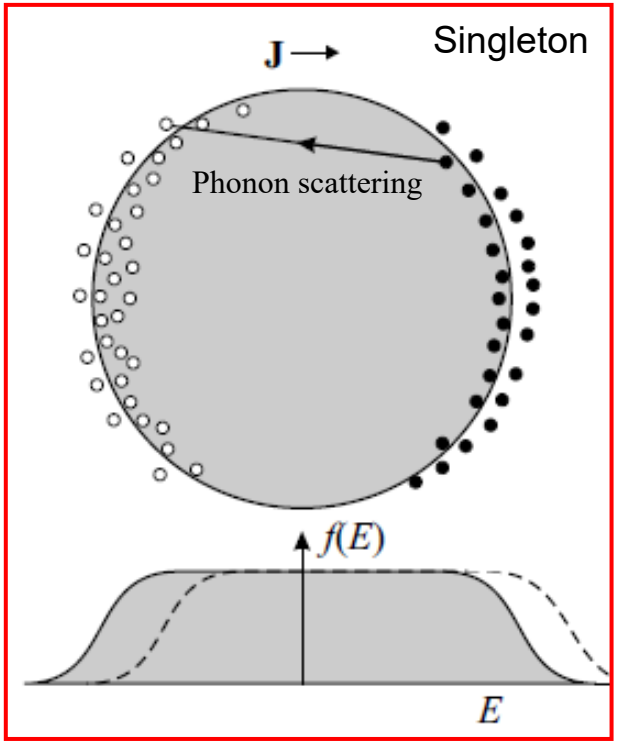
Table 5 Experimental Lorenz numbers Kittel

	$L \times 10^8$ watt-ohm/deg^2			$L \times 10^8$ watt-ohm/deg^2	
Metal	0°C	100°C	Metal	0°C	100°C
Ag	2.31	2.37	Pb	2.47	2.56
Au	2.35	2.40	Pt	2.51	2.60
Cd	2.42	2.43	Su	2.52	2.49
Cu	2.23	2.33	W	3.04	3.20
Mo	2.61	2.79	Zn	2.31	2.33

- As the temperature is lowered the Lorentz number decreases, e.g. for pure Cu at 15K it is an order of magnitude smaller than L_0
- Attributed to a difference between the thermal and electrical scattering times

Electrical transport in metals

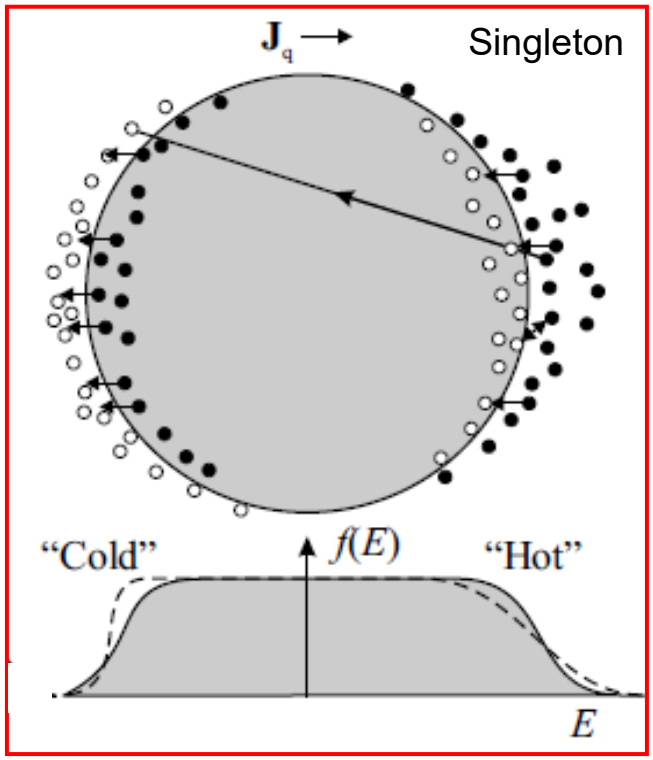
- Previously we have seen that $\mathbf{v}_g = (1/\hbar)\nabla_{\mathbf{k}} E(\mathbf{k})$ - so electron velocities are always perpendicular to the surface of constant energy
- Consider electrical conduction; with no electric field the velocities are in random directions and no net current flows
- When a electric field is applied, electrons at the Fermi surface acquire a small amount of extra velocity in a particular direction - top part of diagram
- Electrons on RHS of fermi surface move into slightly higher energy states, other electrons fill the states vacated. On LHS states are vacated
- Fermi surface moves by a small amount where $\delta k = \frac{1}{\hbar} m^* v_d$, $v_d \approx 10^{-3}\,\mathrm{ms}^{-1}$ is the drift velocity
- Note that $v_d \ll v_F \approx 3\times10^6\,\mathrm{ms}^{-1}$
- The lower diagram shows the Fermi-Dirac distribution for left heading and right heading electrons (dotted) and equilibrium situation (grey)
- τ_σ is time to randomise an electron's forward velocity, a scattering process sends an electron heading to the right into empty state heading left



Black circles filled states, white circles empty states

Thermal transport in metals

- For thermal conductivity
- In kinetic theory, when conducting heat, gas molecules travelling in one direction have a higher thermal energy (or temperature) than those travelling in the other direction
- In the diagram the electrons travelling left are cooler with a less smeared out Fermi-Dirac distribution than those travelling to the right – in the direction of the heat flow
- The long arrow represents phonon scattering events at room temperature which affect both electrical and thermal conduction.



- The short arrows are phonon scattering processes at low temperatures which only affect thermal conduction – by warming up cold electrons and cooling hot electrons
- τ_κ is the time to randomise an electron's thermal energy, a scattering process can cause an electron to lose or gain $\sim k_B T$ of energy and move into an empty state close by or move from hot side of Fermi surface to cold side by scattering off a high momentum phonon

Matthiessens's rule

- We assume that electronic scattering rates are additive

$$\frac{1}{\tau} = \frac{1}{\tau_1} + \frac{1}{\tau_2} + \frac{1}{\tau_3} + \ldots.$$

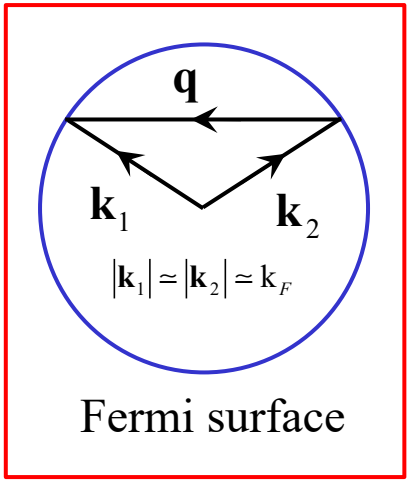
- Where the τ_j^{-1} are scattering rates due to different processes e.g. collisions with or emission/absorption of phonons, scattering from impurities etc
- This equation suggests that the scattering process with the shortest scattering time will dominate – so we can predict regions of temperature where we can ignore all forms of scattering but one
- E.g. it is reasonable to assume that the scattering of electrons at high temperature will be almost entirely due to phonons
- Matthiessen's rule is only an approximation, it fails when the outcome of one scattering process influences another and when one or more scattering time is a function of $\mathbf{k}$
- In the second case κ, σ will involve total scattering times due to all processes averaged over $\mathbf{k}$ whereas the equation implies the summation of reciprocals of each scattering time individually averaged over all $\mathbf{k}$
- An application of Matthiessen's rule is the calculation of electron mobility in two-dimensional electron gases for different scattering processes

Emission and absorption of phonons

- We picture phonons as propagating local distortions of the crystal which may scatter an electron in two different ways:
- *Elastic processes*, where both electron and phonon change wavevector and energy, constrained by conservation of energy and momentum
- *Inelastic processes* where the phonon is emitted or absorbed by an electron causing the electron's wavevector and energy to change
- So far we have assumed that the positions of ions are not affected by the presence of mobile electrons – unrealistic since they are both highly charged
- In reality the passage of an electron will result in the distortion of the lattice around it. The electron can be scattered by this – it has emitted a phonon
- Phonons behave as massless bosons – they can be created and destroyed in a similar way to photons
- They have a black body distribution with energy at temperature T, $\hbar\omega \sim k_B T$
- So when an electron scatters from or absorbs a phonon, the phonon will have an energy $\sim k_B T$
- Electrons are distributed within about $\pm k_B T$ of the Fermi energy. An electron can only emit a phonon up to energies of roughly $k_B T$, since there are no unoccupied states for it to fall into at lower energies

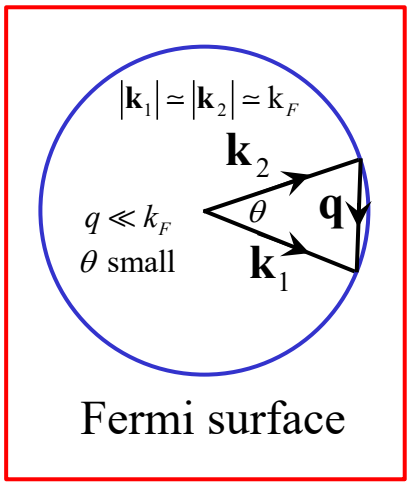
Electron-phonon scattering at room temperature

- The probability of emitting a phonon of energy $\sim k_B T$ will have a similar temperature dependence to the probability of absorbing a phonon since:
- Emission depends on density of available phonon states with energy $\sim k_B T$
- Absorption depends on the number of phonons around with an energy $\sim k_B T$
- The Debye temperature, θ_D of most metals is within about 100K of room temperature (298K) (e.g. Cu 315K, Fe 420K, Al 394K, Pt 230K, Ag 215K)
- $k_B \theta_D$ is roughly the energy of the most energetic phonons, so at room temperature phonons with energy $\hbar\omega \sim k_B \theta_D$ will have wavevectors similar to the width of the Brillouin zone and the Fermi wavevector $q \sim k_F$
- So 1 phonon can scatter an electron to the other side of the Fermi surface
- Hence at room temperature the electrical and thermal scattering rates are approximately equal $\tau_\sigma^{-1} \approx \tau_\kappa^{-1}$ (and hence the Weidemann-Franz law is obeyed) and are proportional to the number of phonons with $\hbar\omega \sim k_B T$



Electron-phonon scattering at low temperatures

- At low temperatures $T \ll \theta_D$ phonons will have energies $k_B T \ll k_B \theta_D$ so $\mathbf{q}$ is much less than the size of the Brillouin zone and the Fermi wavevector
- One inelastic scattering event will be able to change the electrons energy by $\sim k_B T$ and hence the thermal scattering rate $\tau_\kappa^{-1} \propto$ no. of phonons with $\hbar\omega \sim k_B T$ which is $\propto T^3$ at low temperatures from Debye theory
- Hence one phonon scattering event (elastic or inelastic) will be unable to knock the electron to the other side of the Fermi surface (required for electrical scattering) and so the electrical scattering rate is much less than the thermal scattering rate $\tau_\sigma^{-1} \ll \tau_\kappa^{-1}$



- This is the reason for the failure of the Weidemann-Franz law at low T
- To take account of the fact that that many scattering events through a small angle θ are required before the excess forward velocity of the electron is randomised the scattering rate must included a weighting factor

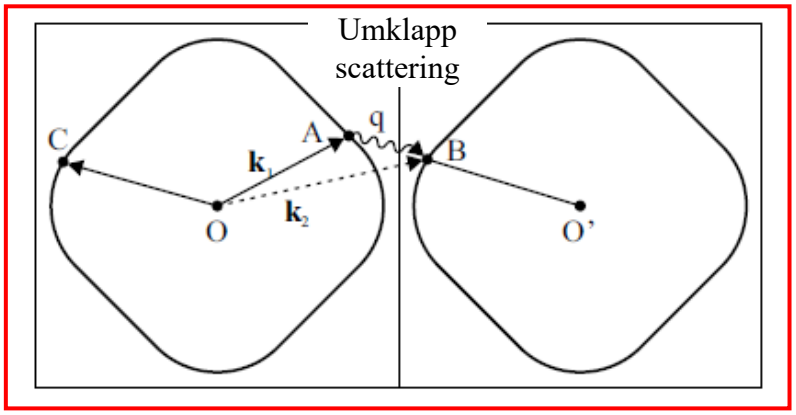
$$1-\cos\theta \approx \theta^2/2 \approx q^2/2k_F^2 \approx \omega^2/2k_F^2 v_\phi^2 \propto T^2$$

where $\omega = v_\phi q$ is the phonon dispersion relation, v_ϕ the speed of sound

- Hence we have $\tau_\sigma^{-1} \propto T^5$, $\tau_\kappa^{-1} \propto T^3$ and so τ_σ^{-1} is much smaller at low T

Electron-phonon scattering at low temperatures

- The $\tau_\sigma^{-1} \propto T^5$ dependence is rarely exactly obeyed
- The periodicity of k-space allows phonons with small q to scatter electrons at the Fermi surface (A) into empty states with energy $\sim E_F$ in an adjacent Brillouin zone (B). These states may have a velocity almost opposite to that of the original state, This is called *umklapp* scattering



- Complicated Fermi surfaces may allow scattering of electrons by phonons with small q into parts of the Fermi surface with very different velocities
- Both of these effects give a scattering rate $\tau^{-1} \propto e^{-\theta_F/T}$ where θ_F is a characteristic temperature depending on the Fermi surface geometry
- For very low temperatures the phonon scattering becomes negligible and scattering of electrons by impurities and defects becomes dominant
- Impurities have a different ionic core from the host metal and therefore will often appear to be charged with respect to the background.
- Scattering of electrons by impurities deflects them through large angles. One scattering event degrades thermal and electrical transport in the same way and $\tau_\kappa = \tau_\sigma$ hence $\sigma \sim \text{const},\ \kappa \propto T$ and the Weidemann-Franz law holds

Scattering – electrical resistivity

- Table gives summary of the temperature dependence of scattering times and electrical and thermal conductivities
- (a) Resistance relative to room temperature value for three samples of Na of different purity. The results show a constant resistance at low temperature determined by impurity scattering. Resistance rises for $T>\sim 10K$

Temperature (scatterer)	Scattering times	κ σ	W-F ratio
Very low (impurities)	$\tau_\kappa \approx \tau_\sigma$ $\sim$ const	$\kappa \propto T$, $\sigma \sim$ const	L_0
$T \sim \theta_D/10$ (phonons)	$\tau_\kappa \propto T^{-3}$, $\tau_\sigma \propto T^{-5} \rightarrow e^{-\theta_F/T}$	$\kappa \propto T^{-2}$, $\sigma \propto T^{-5} \rightarrow e^{-\theta_F/T}$	$< L_0$
$T >\sim \theta_D$ (phonons)	$\tau_\kappa \approx \tau_\sigma$ $\propto T^{-1}$	$\kappa =$const, $\sigma \propto T^{-1}$	L_0

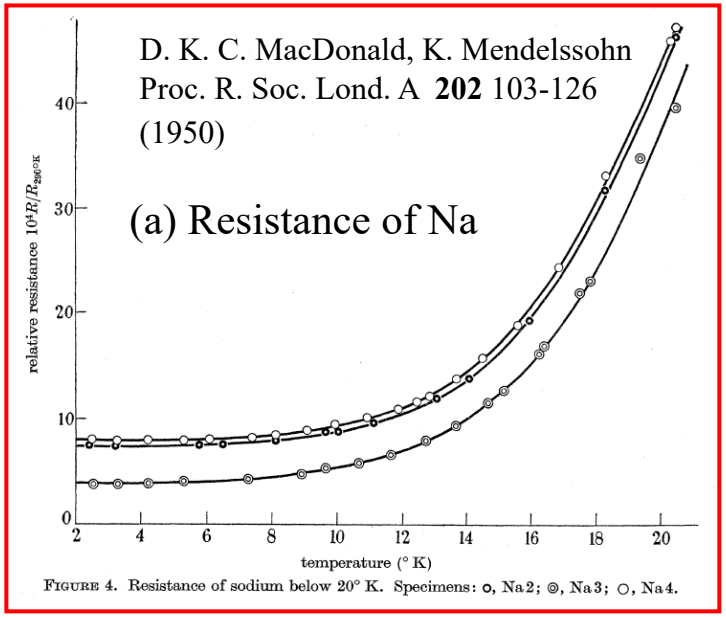
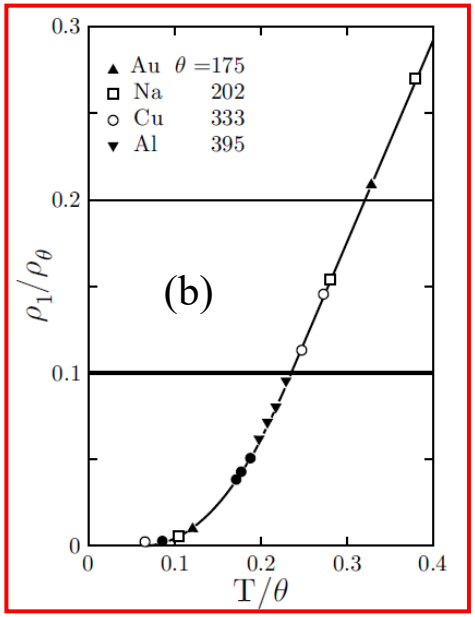


FIGURE 4. Resistance of sodium below 20° K. Specimens: o, Na 2; ◎, Na 3; ○, Na 4.



- (b) shows normalised electrical resistivity data for several metals as a function of normalised temperature. They all fit the same form as a function of temperature
- At low T the resistivity is constant, at high T, $\sigma \propto T^{-1} \Rightarrow \rho \propto T$

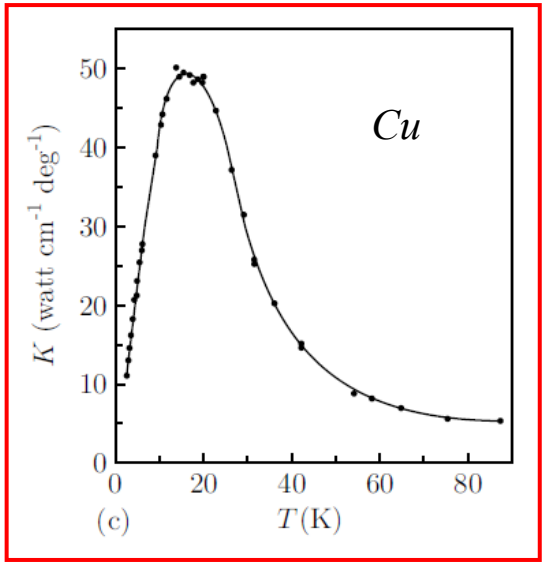
Scattering - thermal conductivity

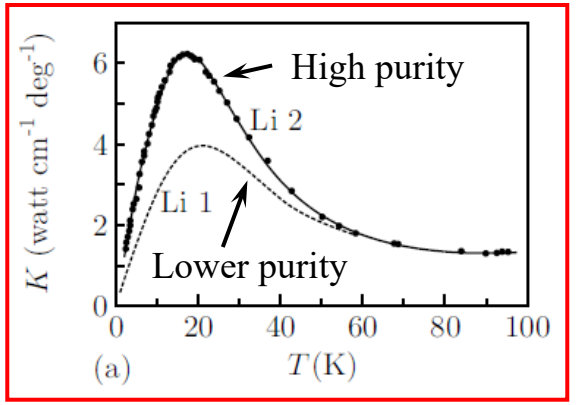
- Thermal conductivity κ rises linearly with T, before reaching a peak then decreasing to a values independent of T
- Peak in κ higher for samples with fewer impurities – confirmed by results for both Li and Na
- Similar results for Cu
- A good illustration of the competition between scattering times

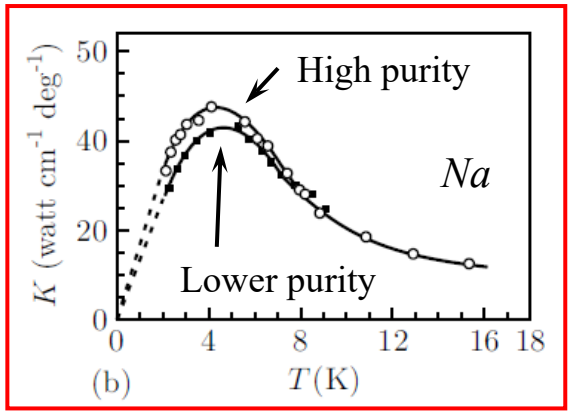
$$\tau^{-1} = \tau_{\text{imp}}^{-1} + \tau_{ph}^{-1}$$

- At low T, few phonons so $\tau_{imp} < \tau_{ph}$, impurity scattering dominates and $\kappa \propto T$. As T rises to $\frac{\theta_D}{10}$ phonon scattering time becomes shorter $\tau_{ph} < \tau_{imp}$ and phonon scattering dominates with $\kappa \propto T^{-2}$

Temperature (scatterer)	Scattering times	κ σ	W-F ratio
Very low (impurities)	$\tau_\kappa \approx \tau_\sigma$ $\sim$ const	$\kappa \propto T$, $\sigma \sim$ const	L_0
$T \sim \theta_D/10$ (phonons)	$\tau_\kappa \propto T^{-3}$, $\tau_\sigma \propto T^{-5} \rightarrow e^{-\theta_F/T}$	$\kappa \propto T^{-2}$, $\sigma \propto T^{-5} \rightarrow e^{-\theta_F/T}$	$< L_0$
$T >\sim \theta_D$ (phonons)	$\tau_\kappa \approx \tau_\sigma$ $\propto T^{-1}$	κ =const, $\sigma \propto T^{-1}$	L_0







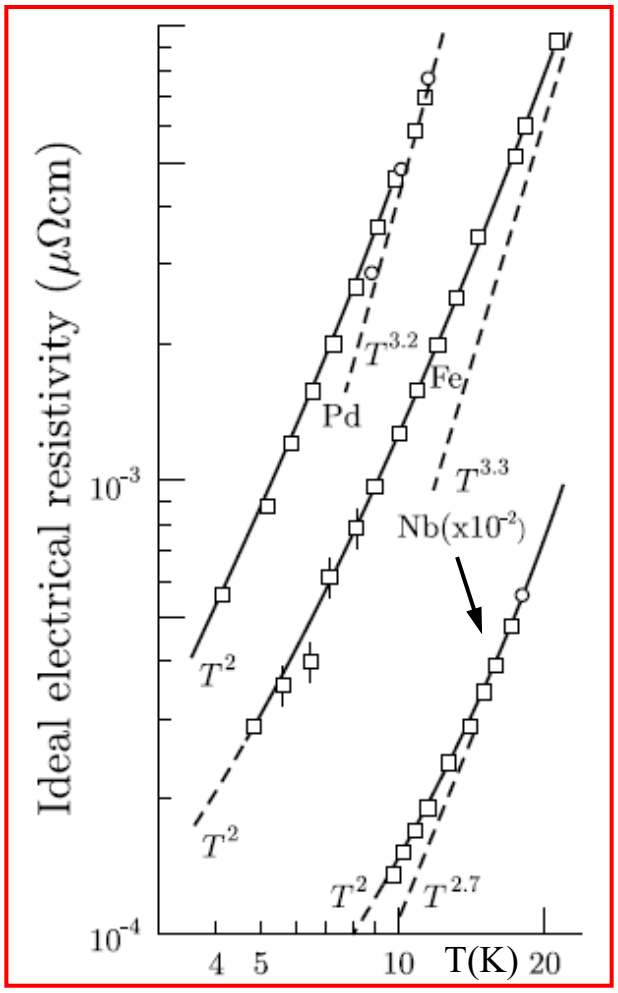
Low temperature solid state physics
H M Rosenberg OUP 1963

Electron-electron scattering

- In metals with simple Fermi surfaces e-e scattering relatively unimportant
- Initial and final state for both electrons have $E \simeq E_F$, $k \simeq k_F$
- Energy and momentum must also be conserved
- This combination makes electron-electron scattering quite unlikely
- It becomes important when (a) the Fermi surface is complicated so conservation of E, k becomes easier (b) density of states at E_F is very large due to large effective mass, increasing number of initial and final states
- Examples: transition metal elements and heavy fermion compounds
- Consider a filled Fermi sphere and a single excited electron with $\epsilon_1 > E_F$
- To be scattered it must interact with an electron with $\epsilon_2 < E_F$ since only states with energies less then the Fermi energy are occupied
- Pauli exclusion principle requires that these electrons can only scatter into unoccupied levels where $\epsilon_3 > E_F$, $\epsilon_4 > E_F$
- Energy conservation requires $\epsilon_1 + \epsilon_2 = \epsilon_3 + \epsilon_4$
- If $\epsilon_1 = E_F$ these conditions can only be fulfilled if $\epsilon_2 = \epsilon_3 = \epsilon_4 = E_F$
- The allowed wavevectors for electrons $2,3,4$ must then occupy zero volume in k-space which makes the probability of this process very small at zero T
- Hence the electron scattering lifetime at $\epsilon = E_F$, $T = 0$ is infinite

Electron-electron scattering

- When ϵ_1 is a little different from E_F some *phase space* becomes available for the process since the other 3 energies can now vary within a shell of thickness $|\epsilon_1 - E_F|$ about the Fermi surface
- This gives a scattering rate $\propto (\epsilon_1 - E_F)^2$ because once ϵ_2, ϵ_3 have been chosen, energy conservation allows no choice for ϵ_4
- If the excited electron is superimposed on a thermal distribution of electrons at non-zero T, there is an additional range of choice in energies available for the scattering process with rate $\propto (k_B T)^2$
- So overall $\tau^{-1} = \alpha(\epsilon_1 - E_F)^2 + \beta(k_B T)^2$ with α, β constant



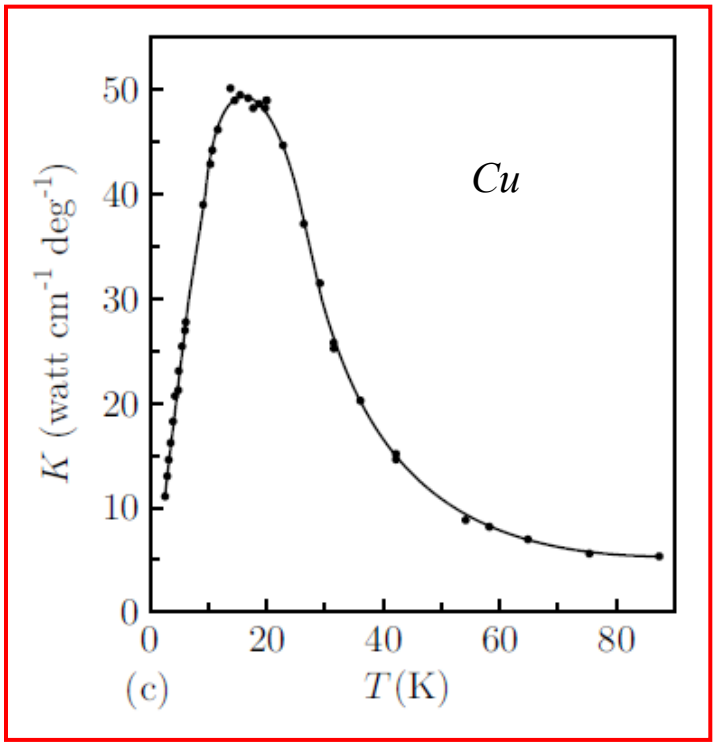
- At finite temperature $\epsilon_1 - E_F \sim k_B T$ so we can say $\tau^{-1} \propto T^2$, $\tau \propto T^{-2}$ and hence $\sigma \propto T^{-2}, \rho \propto T^2$
- Figure shows resistivity of Pd, Fe, Nb with low temperature T^2 dependence visible

Summary of Lecture 13

- Scattering in metals
- Wiedemann-Franz law
- Electrical and thermal transport in metals
- Matthiessens's rule
- Emission and absorption of phonons
- Electron-phonon scattering at room and low temperatures
- The effect of scattering on electrical resistivity and thermal conductivity
- Electron-electron scattering

Quantum Condensed Matter Physics

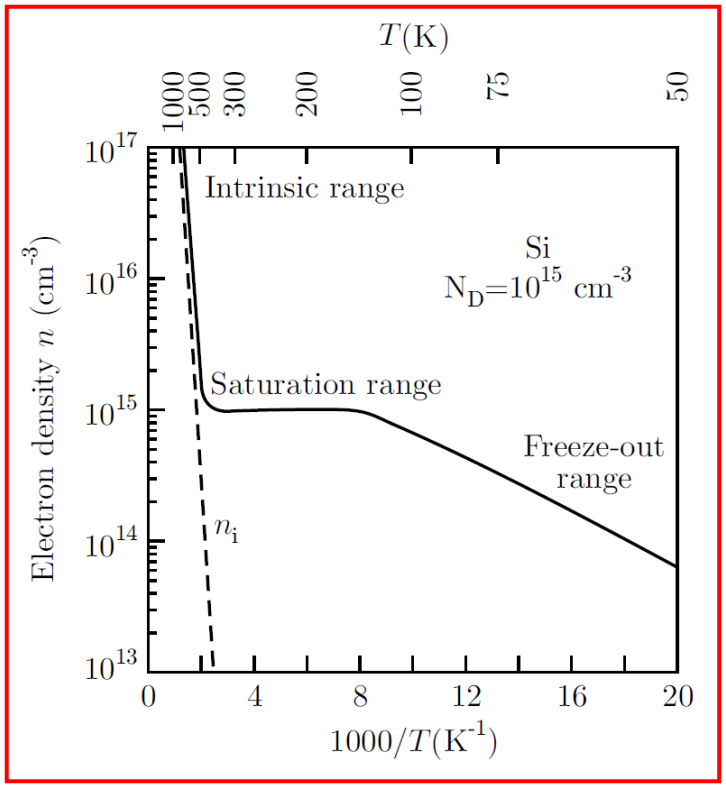
Lecture 13



The end

Quantum Condensed Matter Physics

Lecture 14



David Ritchie

Quantum Condensed Matter Physics

1. Classical and Semi-classical models for electrons in solids (3L)
2. Electrons and phonons in periodic solids (6L)
3. Experimental probes of band structure (4L)
4. Semiconductors and semiconductor devices (5L)

Intrinsic semiconductors, law of mass action, doping in semiconductors, impurity ionisation, variation of carrier concentration and mobility with temperature - impurity and phonon scattering, Hall effect with two carrier types.

Metal to semiconductor contact. P-n junction; charge redistribution, band bending and equilibrium, balance of currents, voltage bias. Light emitting diodes; GaN, organic…….

5. Electronic instabilities (2L)
6. Fermi Liquids (2L)

Semiconductors – intrinsic carriers

- In semiconductors the energy gap is small enough so thermal excitation of the carriers across the gap is important
- Diagram shows density of states for electrons and holes as well as the Fermi function determining occupancy of thermally excited states
- The chemical potential lies mid-gap
- We calculate the thermal intrinsic carrier concentration in a model semiconductor with parabolic electron and hole bands
- The conduction and valence band dispersions are given by

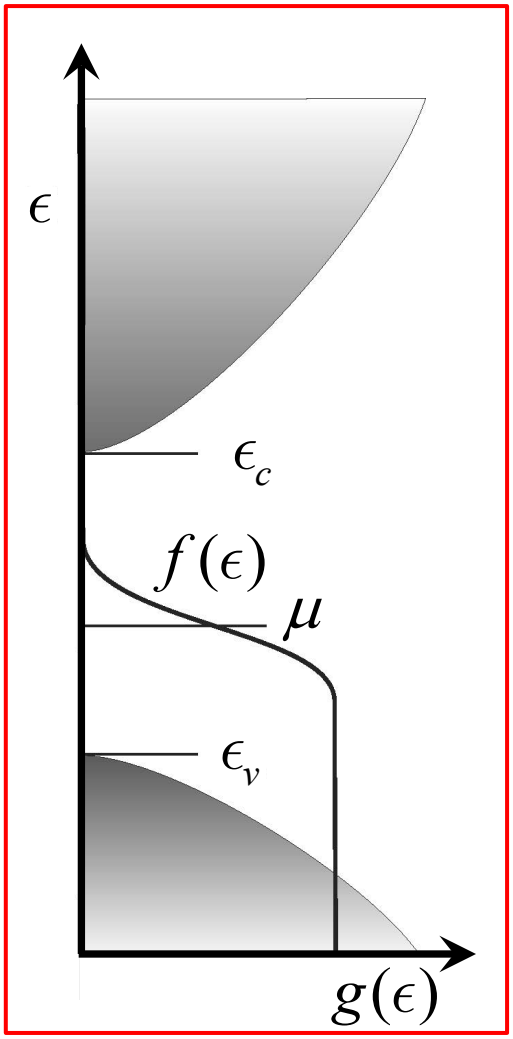
$$\epsilon_c(k) = \epsilon_c + \frac{\hbar^2 k^2}{2m_e^*}, \quad \epsilon_v(k) = \epsilon_v - \frac{\hbar^2 k^2}{2m_h^*}$$

- Density of states for the conduction band

$$g_e(\epsilon) = \frac{1}{2\pi^2}\left(\frac{2m_e^*}{\hbar^2}\right)^{3/2}(\epsilon - \epsilon_c)^{1/2}$$

- Density of states for the valence band

$$g_h(\epsilon) = \frac{1}{2\pi^2}\left(\frac{2m_h^*}{\hbar^2}\right)^{3/2}(\epsilon_v - \epsilon)^{1/2}$$



Semiconductors – intrinsic carriers

- We can calculate the carrier density when the chemical potential μ is known
- For electrons in the conduction band $n = \int_{\epsilon_c}^{\infty} g_e(\epsilon) f(\epsilon) \mathrm{d}\epsilon$ with the Fermi function
$$f(\epsilon) = \frac{1}{e^{(\epsilon-\mu)/(k_B T)} + 1} \approx e^{-(\epsilon-\mu)/(k_B T)}$$
- Where the approximation is valid for $\epsilon - \mu >> k_B T$ - a non-degenerate gas
- Hence
$$n \simeq \frac{1}{2\pi^2}\left(\frac{2m_e^*}{\hbar^2}\right)^{3/2} \int_{\epsilon_c}^{\infty} (\epsilon - \epsilon_c)^{1/2} e^{-(\epsilon-\mu)/(k_B T)} \mathrm{d}\epsilon = 2\left(\frac{m_e^* k_B T}{2\pi\hbar^2}\right)^{3/2} e^{-(\epsilon_c-\mu)/(k_B T)}$$
- And a similar calculation for holes gives
$$p \simeq 2\left(\frac{m_h^* k_B T}{2\pi\hbar^2}\right)^{3/2} e^{-(\mu-\epsilon_v)/(k_B T)}$$
- We define temperature dependent concentrations representing the number of states within~ $k_B T$ of the band edge for the conduction and valence bands
$$n_c(T) = 2\left(\frac{m_e^* k_B T}{2\pi\hbar^2}\right)^{3/2}, \quad n_v(T) = 2\left(\frac{m_h^* k_B T}{2\pi\hbar^2}\right)^{3/2}$$
- Hence
$$n = n_c(T) e^{-(\epsilon_c-\mu)/(k_B T)}, \quad p = n_v(T) e^{-(\mu-\epsilon_v)/(k_B T)}$$
- The concentrations of electrons and holes in terms of the chemical potential

Semiconductors – intrinsic carriers

- From the last slide $n = n_c(T)e^{-(\epsilon_c-\mu)/(k_BT)}$, $p = n_v(T)e^{-(\mu-\epsilon_v)/(k_BT)}$
- Hence we can write $np = n_c(T)n_v(T)e^{-\epsilon_g/(k_BT)}$ where $\epsilon_g = \epsilon_c - \epsilon_v$
- This result is known as the *law of mass action* and is independent of μ - as yet unknown
- In this derivation we have nowhere assumed that the material is intrinsic and the result holds in the presence of impurities and dopants
- The only assumption made is that the distance of the Fermi level from the edge of both bands is large in comparison to k_BT
- A simple kinetic argument shows why np is constant at a given temperature
- Suppose the equilibrium population of electrons and holes is maintained by blackbody radiation
- The photons generate electron-hole pairs at a rate $A(T)$ while $B(T)np$ is the rate of recombination $e+h=photon$

$$\frac{\mathrm{d}n}{\mathrm{d}t} = A(T) - B(T)np = \frac{\mathrm{dp}}{\mathrm{d}t}$$

- In equilibrium $\mathrm{d}n/\mathrm{d}t = \mathrm{d}p/\mathrm{d}t = 0$ hence $np = A(T)/B(T)$ - a constant at a given temperature T

Semiconductors – intrinsic carriers

- Since the product pn is a constant at a given temperature, the introduction of a small amount of a suitable impurity to increase n will decrease p
- This is important because we can reduce the total carrier concentration $n+p$ in an impure crystal by controlled introduction of suitable impurities – known as compensation.
- In an intrinsic semiconductor the number of electrons equals the number of holes and we can write

$$n_i = p_i = \left(n_c(T)\,p_v(T)\right)^{1/2} e^{-\epsilon_g/(2k_BT)}$$

- The intrinsic carrier concentration depends exponentially on $\epsilon_g/(2k_BT)$ Note $\frac{1}{2}\epsilon_g$ and not ϵ_g - because the creation of an electron also creates a hole
- From a previous slide

$$n \simeq 2\left(\frac{m_e^* k_B T}{2\pi\hbar^2}\right)^{3/2} e^{-(\epsilon_c-\mu)/(k_BT)}, \quad p \simeq 2\left(\frac{m_h^* k_B T}{2\pi\hbar^2}\right)^{3/2} e^{-(\mu-\epsilon_v)/(k_BT)}$$

- If we set $n = p$

$$e^{2\mu/(k_BT)} = (m_h^*/m_e^*)^{3/2} e^{\epsilon_g/(k_BT)} \Rightarrow \mu = \tfrac{1}{2}\epsilon_g + \tfrac{3}{4}k_BT\ln(m_h^*/m_e^*)$$

- Which gives the position of the chemical potential.
- If $m_h^* = m_e^* \Rightarrow \mu = \frac{1}{2}\epsilon_g$ the chemical potential is in the middle of the bandgap

Doped semiconductors

- Carriers can be created in semiconductors by adding impurity atoms – a process known as doping
- Consider effect on a GaAs crystal of replacing a Ga atom by a Si atom
- Si provides 4 electrons instead of the normal 3 from Ga so it appears like a Ga atom with an extra electron and an extra positive charge in the nucleus
- Suppose the electron wanders away from the impurity site, it will experience an attractive force from the charged Si impurity
- The donor energy levels can be calculated as for a hydrogen atom
- We take into account the influence of the surrounding material by making two corrections

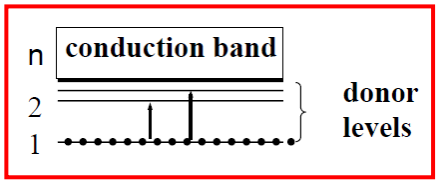
  (1) The Coulomb potential is screened by the dielectric constant, so it is much weaker than in free space (for GaAs $\varepsilon \simeq 13.1$)

  (2) Use effective mass of the electron (for GaAs $m_e^* / m_e = 0.067$)
- The net effect is that the binding energy for the 1s state is now

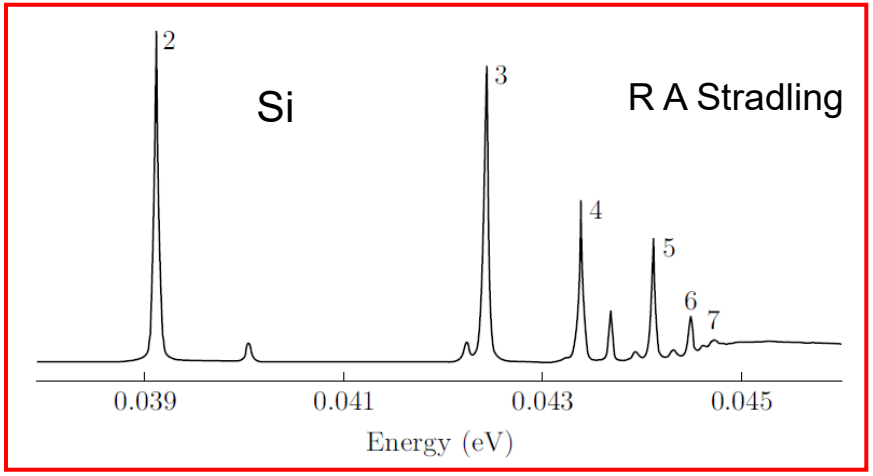
$$\Delta_d = \frac{e^4 m_e^*}{2(4\pi\varepsilon\varepsilon_0\hbar)^2} = \frac{m_e^* / m_e}{\varepsilon^2} \times 13.6 \text{ eV}$$

- Which for GaAs is 5.3meV – much smaller than the bandgap of 1.4eV

Doped semiconductors



- From the last slide $\Delta_d = m_e^* / (\varepsilon^2 m_e) \times 13.6\ \text{eV}$
- For GaAs the Si donor ionisation energy of about 5.3meV is equivalent to a temperature of 50K. So Si donors in GaAs will all be ionised at room temperature
- The figure shows far infra-red absorption due to P impurities in Si (ionization energy ~45meV). The peaks correspond to hydrogen like transitions between the n=1 ground state and higher levels



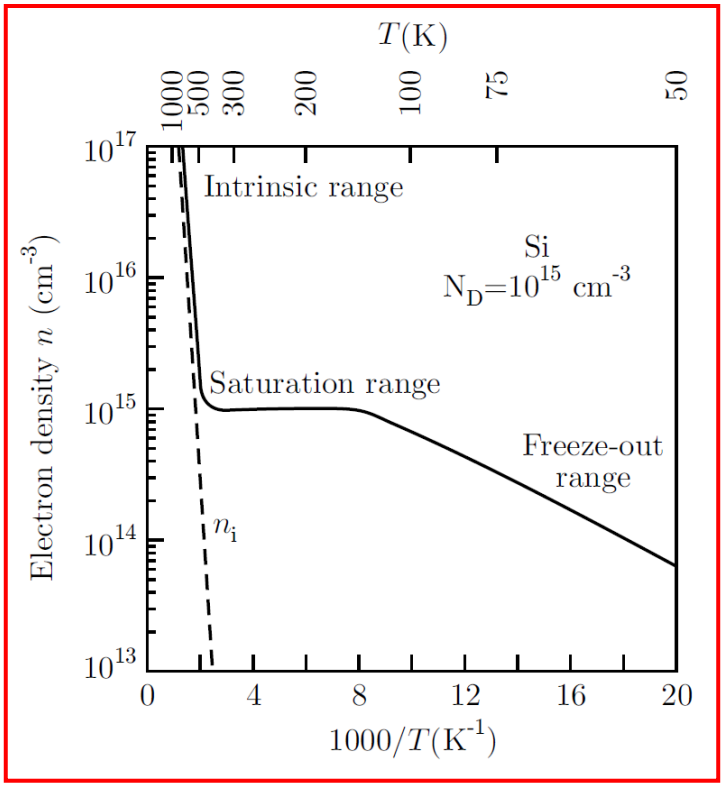
- The hydrogen-like bound states are referenced to the bottom of the conduction band, because the electron unbinds from the donor atom by occupying a conduction band state
- A Beryllium atom in GaAs acts as an acceptor dopant if it sits on a Ga site.
- Be only donates 2 electrons as opposed to the normal 3 from Ga (donating a hole) and appears like a Ga atom with a negative charge
- So we have a positively charged hole circling a negatively charged nucleus
- The hydrogenic binding energy can be calculated in the same way as for donors – taking into account the hole effective mass

Doped semiconductors

- When the hole unbinds from its Be atom the impurity accepts an electron from the valence band
- The accepted electron is used to complete the covalent bonding with neighbouring atoms and renders the site negatively charged
- So while ionising a donor atom releases an electron into the conduction band, ionising an acceptor atom absorbs an electron from the valence band creating a hole in the valence band
- Even for very low densities of impurities, since the donor or acceptor energies are much smaller than the band gap, impurities are often the main source of electrically active carriers
- If donors predominate the carriers are mostly electrons and the material is said to be n-type.
- If acceptors dominate carriers are mostly holes and the material is p-type
- In most materials there are both donors and acceptor impurities – in GaAs Si on a Ga site is a donor and on an As site is an acceptor, whether you get n or p-type depends on the crystal growth technique
- Experimentally the different carrier regimes may be distinguished by measuring the Hall effect, the sign of which depends on the carrier type

Doped semiconductors – Impurity ionisation

- As long as the no. of donors/acceptors is low enough so the chemical potential lies in the bandgap then the law of mass action holds
- Given the densities of ionised donors and acceptors N_D, N_A we can use $np \simeq 4\left(\frac{k_B T}{2\pi\hbar^2}\right)^3 \left(m_e^* m_h^*\right)^{3/2} e^{-E_g/(k_B T)}$ and the conservation law $n - p = N_D - N_A$ to find values for n, p if we know the effective masses and the bandgap
- Diagram shows the temperature dependence of the electron density in Si with a net donor density $N_D - N_A = 10^{15}\,\mathrm{cm}^{-3}$
- At T<100K the extrinsic electrons freeze out onto the donors, the gradient depends on the donor ionisation energy
- For 150K<T<300K all of the donors are ionised – the saturation range, n is constant
- For T>500K the intrinsic contribution to n becomes larger than extrinsic contribution, the gradient depends on the main bandgap
- The intrinsic contribution, n_i is around $3\text{x}10^9\text{cm}^{-3}$ at RT – negligible



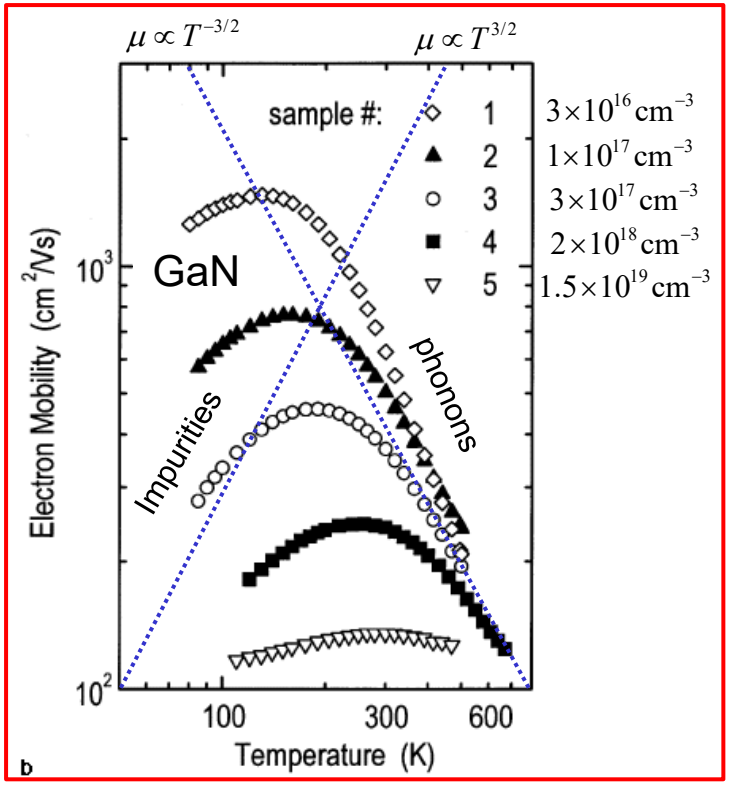
Electrical conductivity with two carrier types

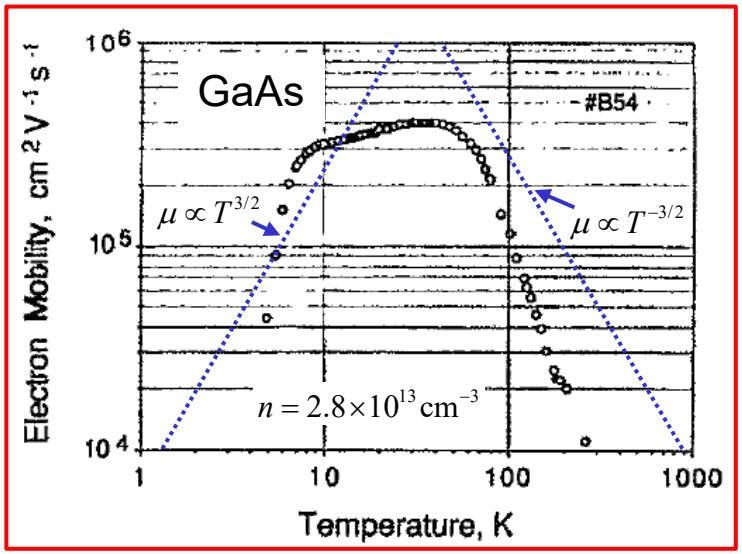
- The electrical conductivity of a semiconductor is a sum of contributions from all carrier types, usually electrons and heavy holes written in terms of carrier densities and mobilities $\sigma = ne\mu_e + pe\mu_{hh}$
- We write electron and hole mobilities in terms of scattering times for electrons and heavy holes $\mu_e = e\tau_e / m_e^*, \quad \mu_{hh} = e\tau_{hh} / m_{hh}^*$
- The temperature dependency of the electrical conductivity is determined by convolutions of the temperature dependences of the carrier concentrations and scattering times. There are two important types of scattering:
- *Impurity scattering* is similar to Rutherford scattering. The scattering crossection varies as ϵ^{-2} and since in the non-degenerate case $\epsilon \sim k_B T$ the crossection varies as T^{-2} and the mean free path as $\lambda \propto T^2$
- The carrier speed $v \propto \epsilon^{1/2} \propto T^{1/2}$ hence scattering time $\tau_{imp} = \lambda / v \propto T^{3/2}$
- Contrast to metals where τ_{imp} is independent of T - carriers in metals have similar energy, but in semiconductors a Boltzmann energy distribution
- *Phonon scattering* with $T \sim \theta_D$ - the number of phonons is $\propto T$ so $\lambda \propto T^{-1}$ and with $v \propto \epsilon^{1/2} \propto T^{1/2}$ we get $\tau_{pho} = \lambda / v \propto T^{-3/2}$
- As a consequence impurity scattering dominates the mobility at low T and phonon scattering at high T with a peak in mobility in between

Electron mobility versus temperature

- Top figure shows electron mobility of 5 samples of GaN with different doping levels and carrier concentrations
- For high temperature variation close to expected $\mu \propto T^{-3/2}$ due to phonon scattering
- At low temperature drop off slower than expected $\mu \propto T^{3/2}$ for impurity scattering – measurements at lower temperatures may reach this dependence
- Lower figure shows highest mobility sample of GaAs ever reported
- At high T close to predicted variation for phonon scattering
- At low T faster drop off than expected – possibly due to *metal-insulator transition*

D Steigerwald et al, JOM **49**, 18 (1997)





C R Stanley et al Appl Phys Lett **58**, 478 (1991)

Hall effect with two carrier types

- Earlier in the course we studied the Hall effect for a single carrier type
- Assuming $\mathrm{d}\mathbf{j}/\mathrm{d}t = -\mathbf{j}/\tau + \left(ne^2/m\right)\left(\mathbf{E}+\mathbf{v}\times\mathbf{B}\right)$
- in the steady state - for electrons

$$\mathbf{j}_e = \left(ne^2\tau_e/m_e\right)\left(\mathbf{E}+\mathbf{v}_e\times\mathbf{B}\right) = ne\mu_e\left(\mathbf{E}+\mathbf{v}_e\times\mathbf{B}\right)$$

- If both electrons and holes are present we have

$$\mathbf{j} = ne\mu_e\left(\mathbf{E}+\mathbf{v}_e\times\mathbf{B}\right) + pe\mu_h\left(\mathbf{E}+\mathbf{v}_h\times\mathbf{B}\right)$$

- Assume current is flowing in the x-direction, current in the y-direction is zero, B is in the z-direction, v_{xe}, v_{xh} are opposite signs and $\mu = v/E$

$$j_x = eE_x(n\mu_e + p\mu_h),\quad 0 = eE_y(n\mu_e + p\mu_h) - eB(n\mu_e v_{ex} + p\mu_h v_{hx})$$

$$\Rightarrow 0 = eE_y(n\mu_e + p\mu_h) + eBE_x(n\mu_e^2 - p\mu_h^2)$$

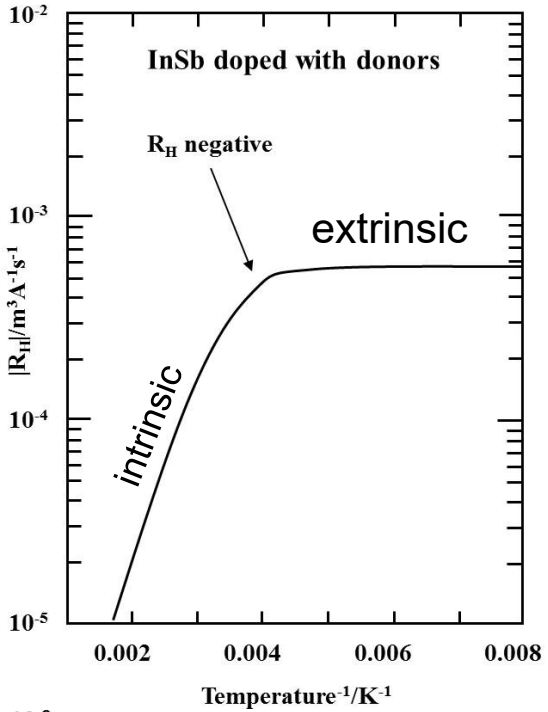
- Eliminating E_x we get

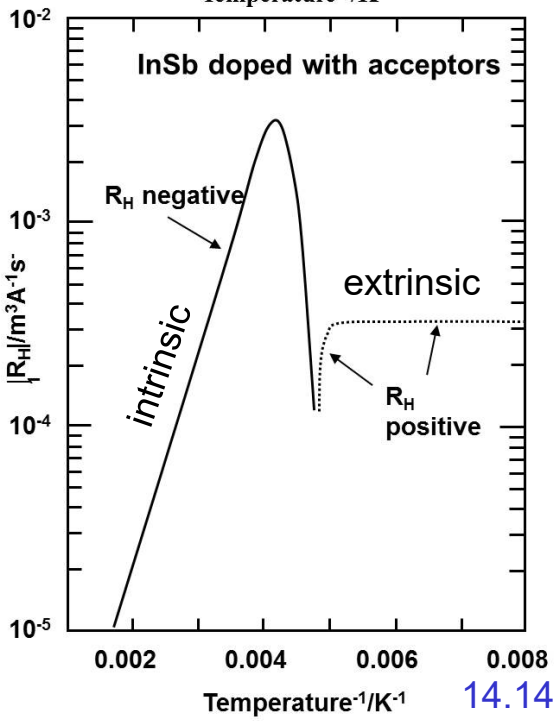
$$E_y = -\frac{j_x B(n\mu_e^2 - p\mu_h^2)}{e(n\mu_e + p\mu_h)^2} \Rightarrow R_H = \frac{E_y}{j_x B} = -\frac{(n\mu_e^2 - p\mu_h^2)}{e(n\mu_e + p\mu_h)^2}$$

- A minority carrier can determine the Hall coefficient sign if the mobility is high enough – e.g. GaAs has $\mu_e = 8800\mathrm{cm}^2V^{-1}s^{-1}$, $\mu_h = 400\mathrm{cm}^2V^{-1}s^{-1}$ at room temperature so electrons are likely to dominate

Hall effect with electrons and holes

- Diagrams show Hall coefficient R_H in InSb plotted as a function of T^{-1}
- Upper diagram - InSb is doped with donors, lower diagram doped with acceptors
- Ratio of mobilities $\mu_e / \mu_h \sim 100$
- At high T InSb is in the intrinsic regime, electrons dominate R_H and the slope of the Hall coefficient can be used to determine the bandgap giving a value of 0.24eV
- At low T in the upper diagram, no intrinsic carriers $R_H = -\frac{1}{ne}$ remains negative giving an electron concentration $n = 1.1\times10^{16}\,\text{cm}^{-3}$
- In the lower diagram as the temperature is lowered the intrinsic electrons freeze out they no longer dominate R_H and R_H changes sign
- In the extrinsic region R_H is positive indicating holes $R_H = \frac{1}{pe} \Rightarrow p = 2\times10^{16}\,\text{cm}^{-3}$



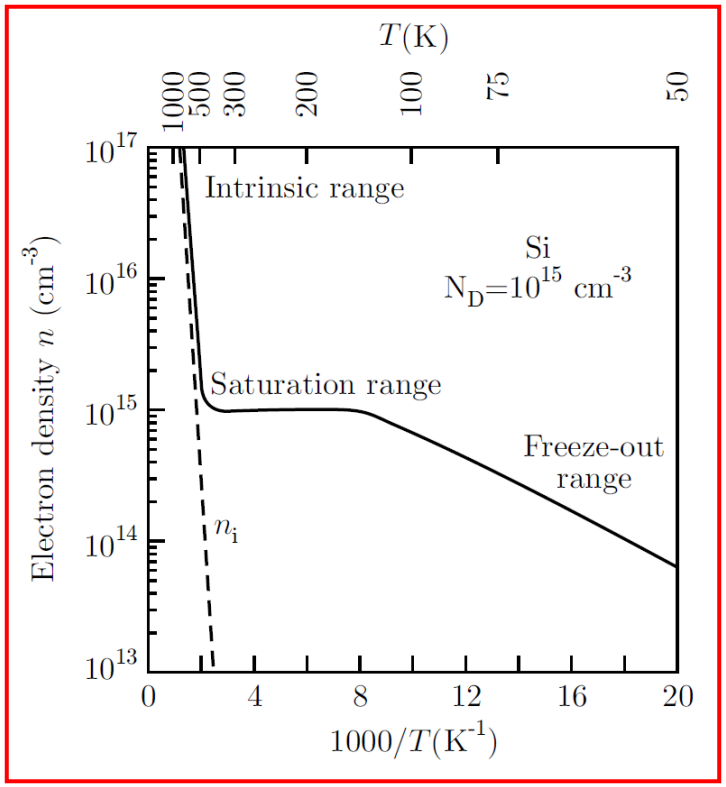


Summary of Lecture 14

- Intrinsic semiconductors, law of mass action
- Doping in semiconductors
- Variation of carrier concentration with temperature, impurity ionisation
- Electrical conductivity, impurity and phonon scattering
- Electron mobility versus temperature
- Hall effect with two carrier types – example InSb

Quantum Condensed Matter Physics

Lecture 14



The end

Quantum Condensed Matter Physics

Lecture 15

David Ritchie

Quantum Condensed Matter Physics

1. Classical and Semi-classical models for electrons in solids (3L)
2. Electrons and phonons in periodic solids (6L)
3. Experimental probes of band structure (4L)
4. Semiconductors and semiconductor devices (5L)

Intrinsic semiconductors, law of mass action, doping in semiconductors, impurity ionisation, variation of carrier concentration and mobility with temperature - impurity and phonon scattering, Hall effect with two carrier types.

Metal to semiconductor contact. P-n junction; charge redistribution, band bending and equilibrium, balance of currents, voltage bias. Light emitting diodes; GaN, organic.

Photovoltaic solar cell; Shockley-Queisser limit, efficiencies…….

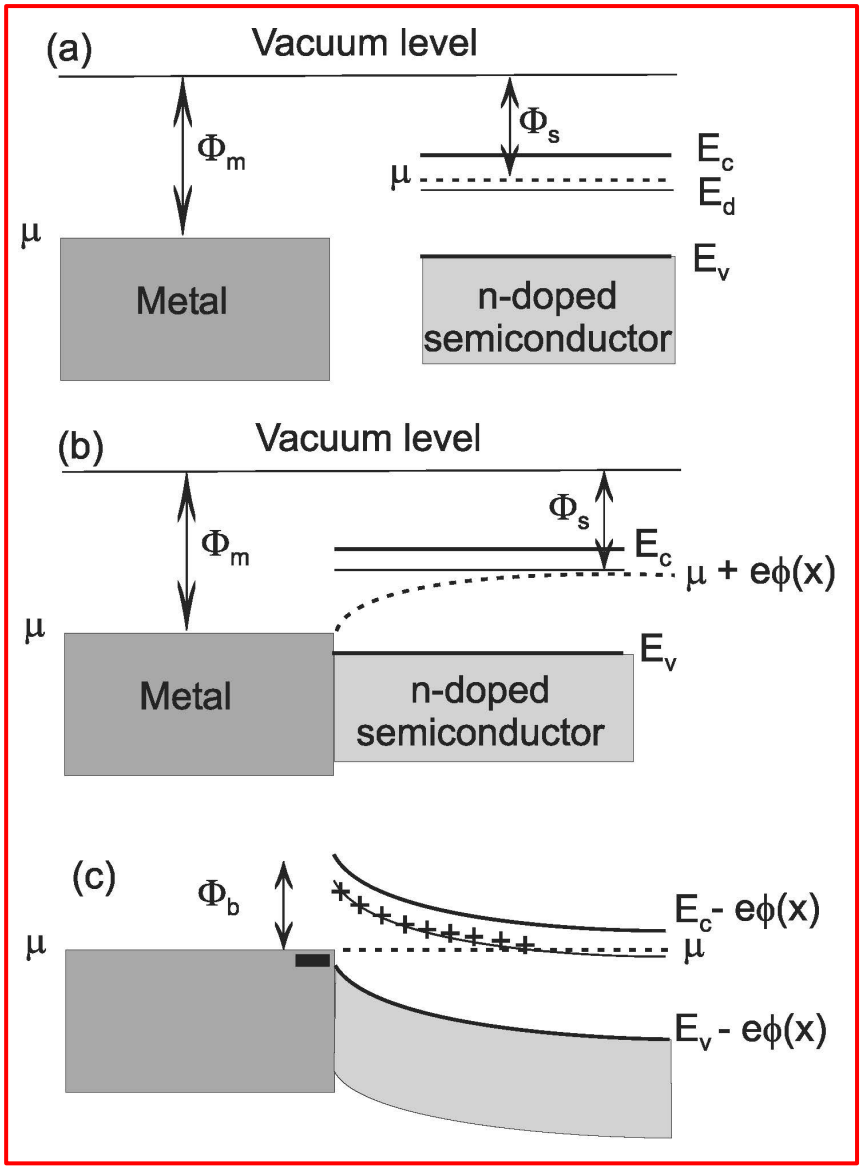
5. Electronic instabilities (2L)
6. Fermi Liquids (2L)

Semiconductor devices

- We consider the general properties of surfaces and interfaces between materials with applications to semiconductor devices
- We use the semiclassical approximation, treating electrons as classical particles with Hamiltonian $H = E_n(\mathbf{k}) - e\phi(\mathbf{r})$, momentum $\mathbf{p} = \hbar\mathbf{k}$ and a spatially varying potential $\phi(\mathbf{r})$
- The potential arises from external applied fields, charges induced by doping and changes in the material composition
- When discussing narrow quantum wells we will need to modify this approximation to quantise the energy levels
- For an isolated solid in equilibrium the energy difference between the chemical potential, μ and the vacuum level is the work function Φ - the energy required to remove an electron from the Fermi level and place it in a state of zero kinetic energy in free space
- 2 different isolated materials with different Φ will have different μ
- When placed in contact their chemical potentials must equalise. Electrons flow to charge the more electronegative material, its potential changes and an overall balance will be established
- In general there will be internal inhomogeneous electric fields

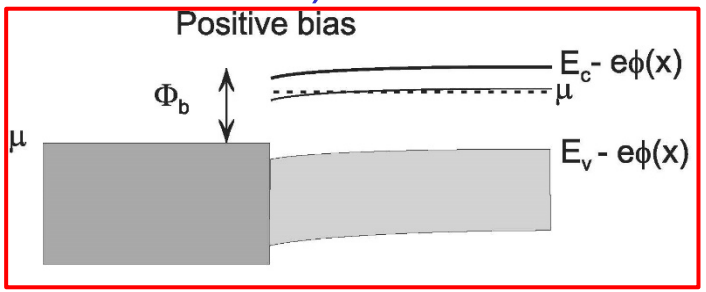
Metal to semiconductor contact

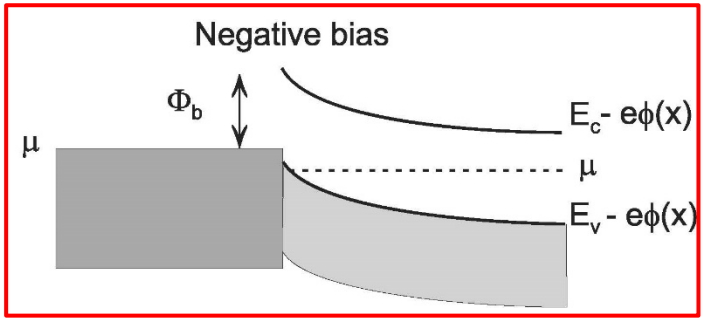
- Consider this process for ideal metal in contact with a semiconductor
- (a) metal and semiconductor not in contact – in equilibrium with vacuum level. n-type semiconductor, μ close to conduction band edge
- (b) In contact, electrons transferred from semiconductor to metal producing electric potential $\phi(x)$ which eventually equilibrates so μ is a constant over whole system, combined function $\mu + \phi(x)$ called *electrochemical potential*
- (c) energy level diagram relative to constant chemical potential. Semiconductor band bends upwards, donor levels emptied of electrons leaving +ve charged depletion region and Schottky barrier Φ_b

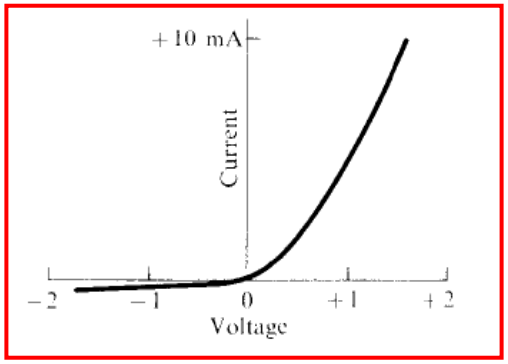


Metal to semiconductor contact

- Barrier set up between metal and semiconductor inhibits current flow.
- An electron must either tunnel through barrier (at low T) or be thermally excited over it (thermionic emission)
- When a large enough bias is applied junction may act as a rectifier, (with a bias applied μ is different for the metal and semiconductor)
- Upper diagram, applying a +ve voltage to the metal relative to the semiconductor lowers the barrier for electrons to enter the metal. Can eventually tip the electron bands so much that the barrier disappears
- Current grows rapidly as +ve bias increases
- Middle diagram, if the metal bias is at a negative voltage relative to the semiconductor the depleted region grows in width and current remains small
- Lowest diagram shows IV characteristic of Silicon-tungsten diode for high frequency (mm wave) rectification

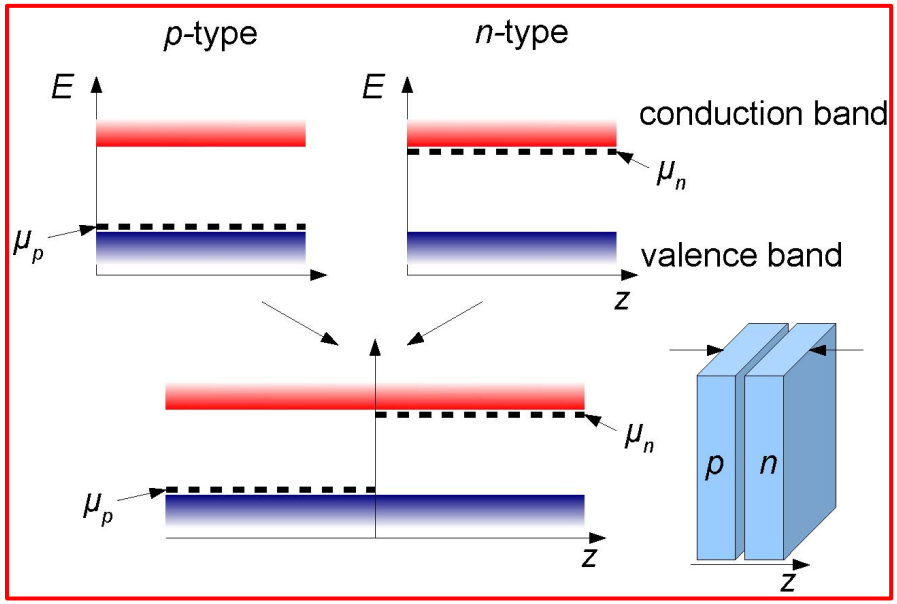


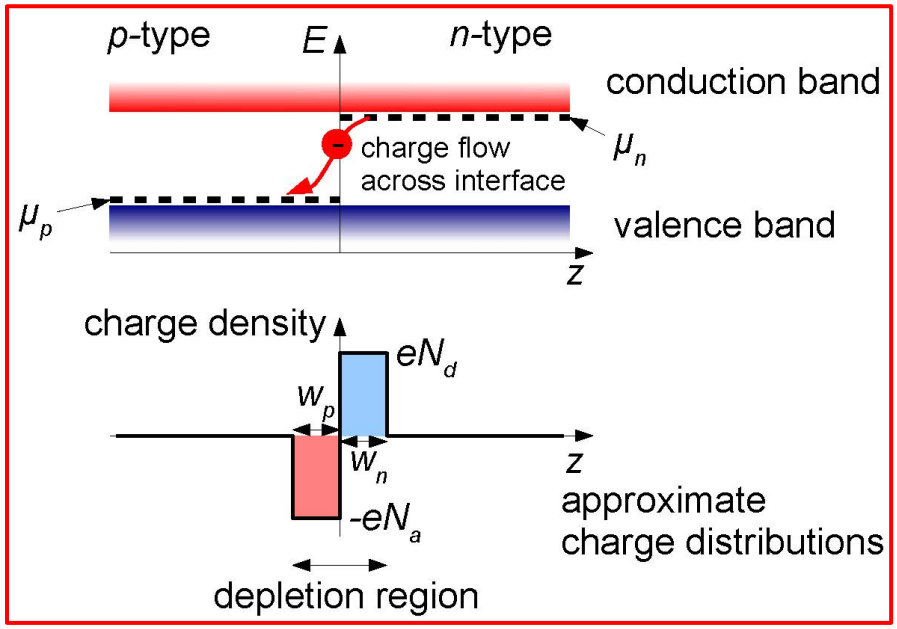




p-n junction – charge redistribution

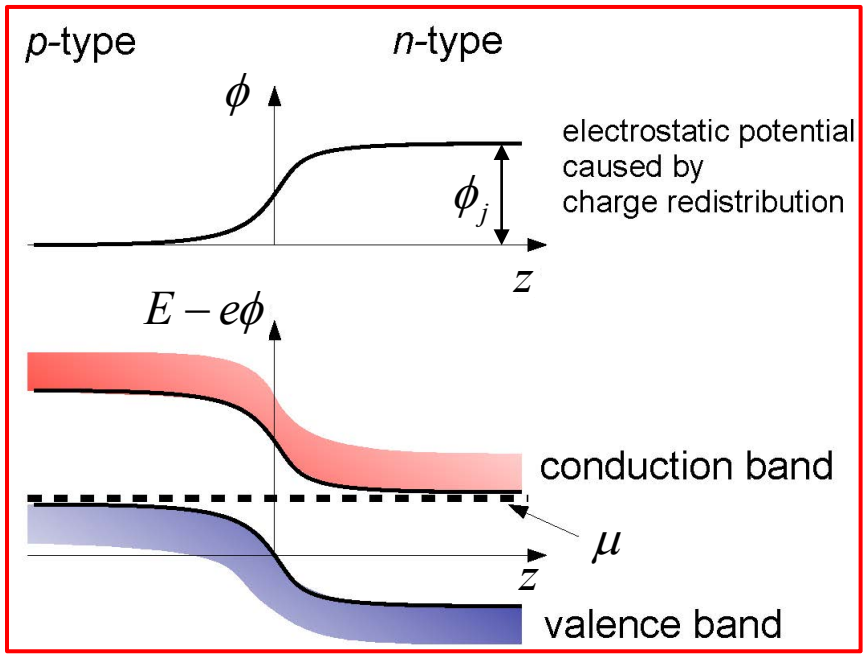
- A p-n junction is formed by inhomogeneous doping when a layer of n-type material is placed next to p-type material
- Inside n-type μ_n just below bottom of conduction band
- Inside p-type μ_p just above top of valence band
- Joining p-n gives step in μ
- Current flows because of different chemical potentials, electrons from n-side fill holes on p-side
- No mobile charges left in *depletion region* around junction – a thickness of between 10nm and 1μm
- Ionised donors have +ve charge on n-side, ionised acceptors have –ve charge on p-side
- Charge neutrality $\Rightarrow N_a w_p = N_d w_n$





p-n junction – band bending

- Poisson's law – charge redistribution causes electrostatic potential
- Energy levels shift $E \rightarrow E - e\phi(z)$
- Far away from junction in n-type μ is close to top of bandgap and in p-type it is close to bottom of band gap
- Charge flows until energy levels have moved enough for μ to line up across junction and equilibrium is reached
- Junction potential $e\phi_j = \mu_n - \mu_p \simeq E_g$
- Charge neutrality and Poisson's equation lead to expressions relating junction potential, ϕ_j thicknesses of n- and p-type depletion regions w_n, w_p and doping densities of n- and p-type regions N_d, N_a



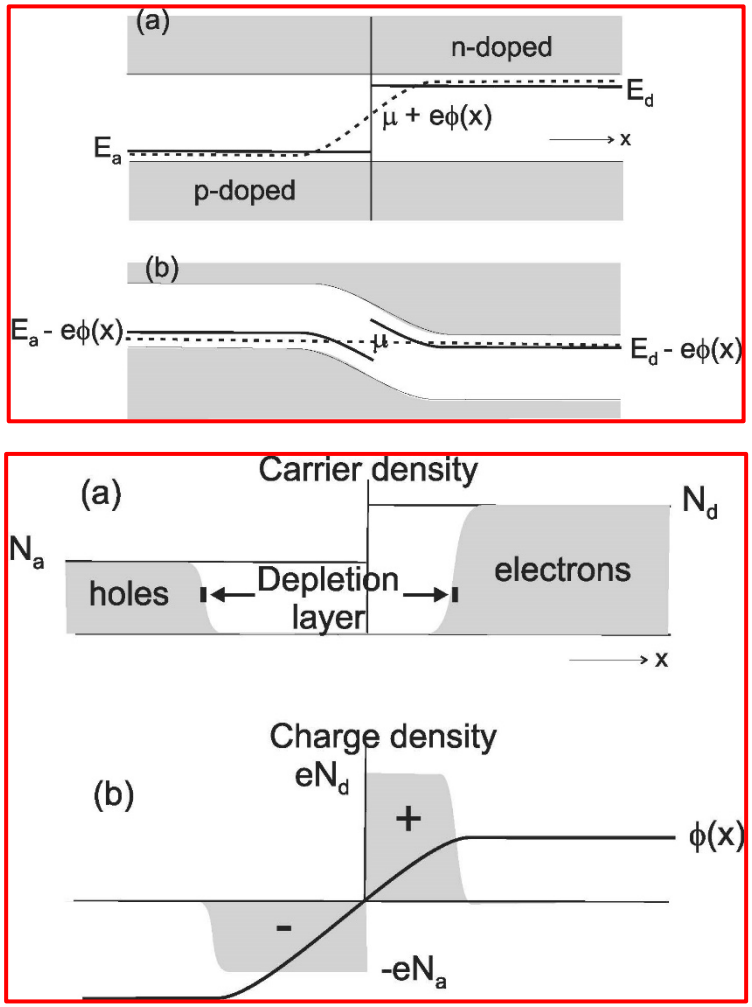
$$N_{(a,d)} w_{(p,n)} = \left(\frac{2\varepsilon_0 \varepsilon \phi_j}{e} \frac{N_a N_d}{N_a + N_d} \right)^{1/2}$$

$$\phi_j = \frac{e}{2\varepsilon_0 \varepsilon} \left(N_a w_p^2 + N_d w_n^2 \right)$$

$$w_p + w_n = \left(\frac{2\varepsilon_0 \varepsilon \phi_j}{e} \frac{N_a + N_d}{N_a N_d} \right)^{1/2}$$

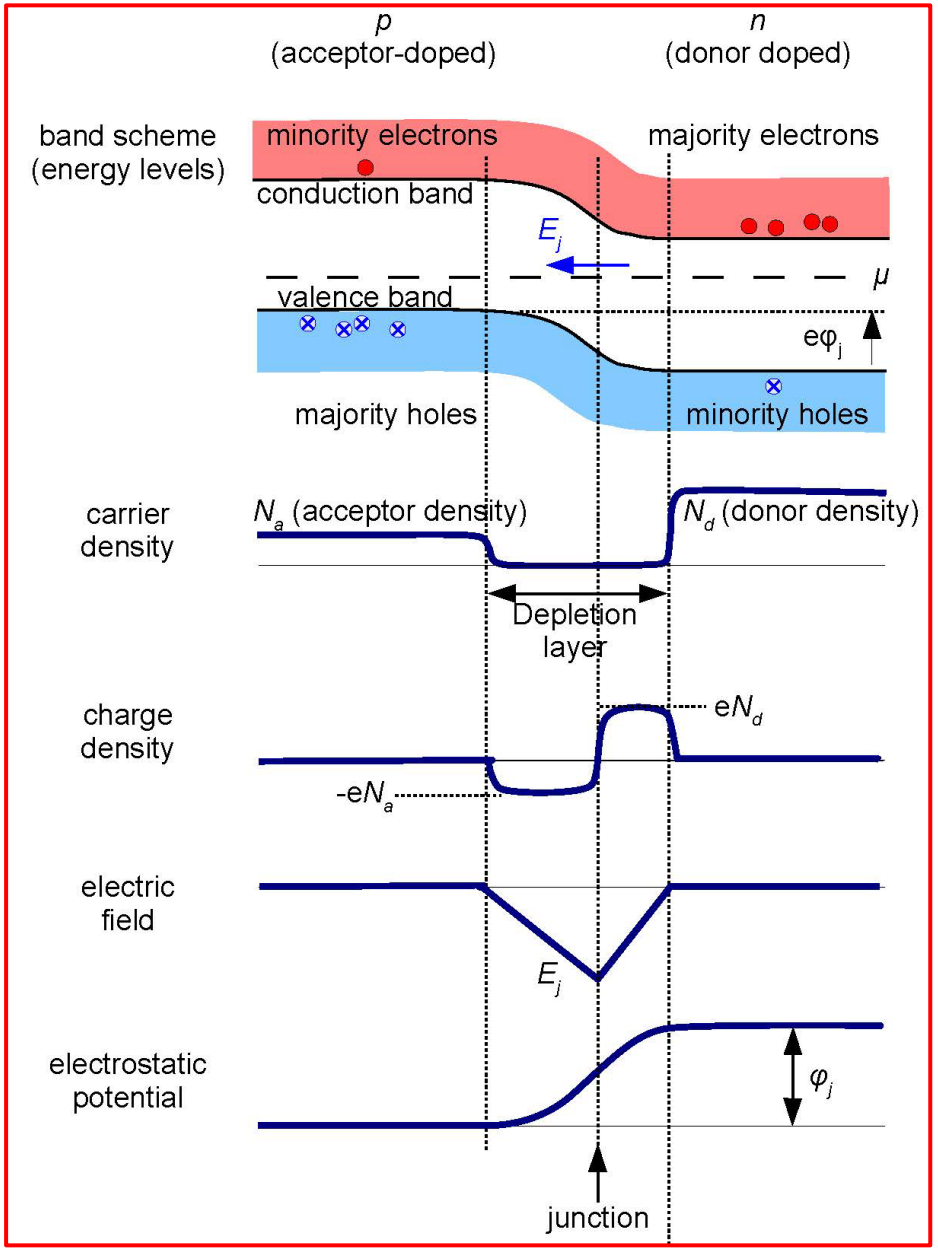
p-n junction in equilibrium

- Top: 2 ways of representing energy levels in a p-n junction (a) includes electrostatic potential in the electrochemical potential (b) chemical potential constant and potential shifts energy levels
- When the donor or acceptor levels pass through μ levels are ionised and annihilate – impurity levels now charged
- Bottom: (a) carrier and (b) charge densities near the depletion region of a p-n junction
- When T is low the carrier density changes abruptly at the point where μ passes through the donor or acceptor level
- Close to the barrier the carriers are depleted – the system is physically charged
- Charge density of $+eN_d$ on the n-type side and $-eN_a$ on the p-type side
- This dipole layer produces a potential $\phi(x)$ shown in (b)
- Potential self consistently determines charge flow and depletion region width



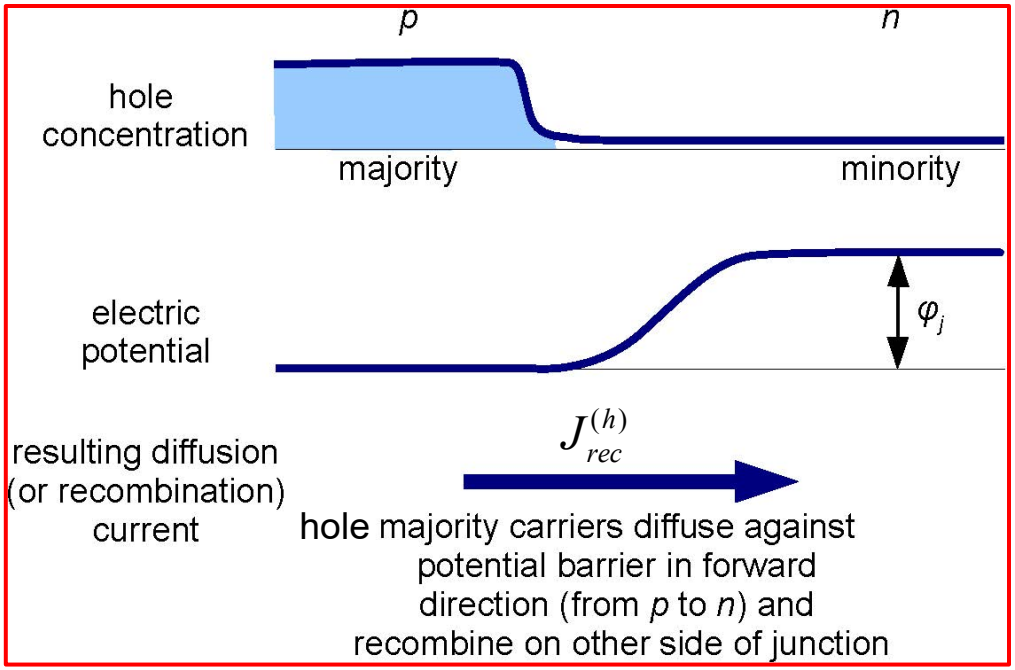
p-n junction in equilibrium - summary

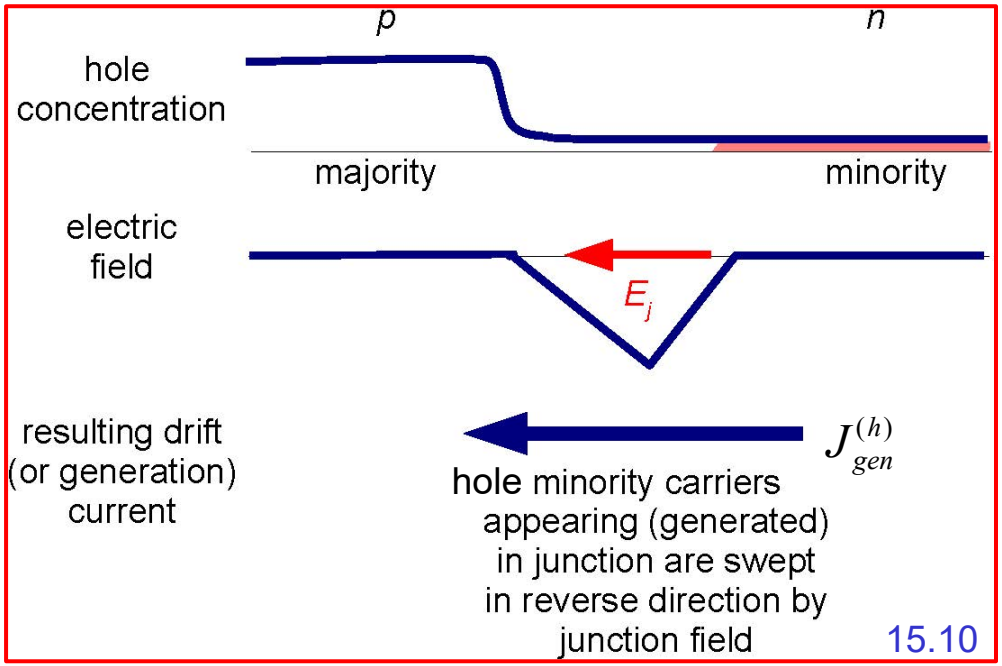
- Overview of p-n junction in equilibrium
- Mismatch in μ causes charge transfer across junction building contact potential, resulting in band bending until μ equal on both sides
- Electrons from n-side cross junction annihilating holes causing carrier free depletion region
- Charge transfer results in space charge, maximum charge density given by dopant concentration
- Space charge causes in-built junction field E_j and contact potential φ_j which builds until charge transfer stops



p-n junction – balance of currents

- *Majority* holes diffuse from p to n-side, recombine with electrons
- Hole *recombination or diffusion* current $J_{rec}^{(h)} = J_0^{(h)} e^{-e\phi_j/(k_B T)} \propto$ probability that holes climb the potential barrier, $J_0^{(h)}$ constant
- Electrons do the same in the opposite direction. Currents add
- Small no. of *minority* holes $(np = n_i^2)$ thermally generated on n-side drift to p-side under influence of in-built field.
- Resulting hole *drift* or *generation* current $J_{gen}^{(h)}$ depends on T, details of band structure and doping
- Electrons do same in opposite direction. Currents add
- Equilibrium: $J_{tot}^{(h)} = J_{rec}^{(h)} - J_{gen}^{(h)} = 0$





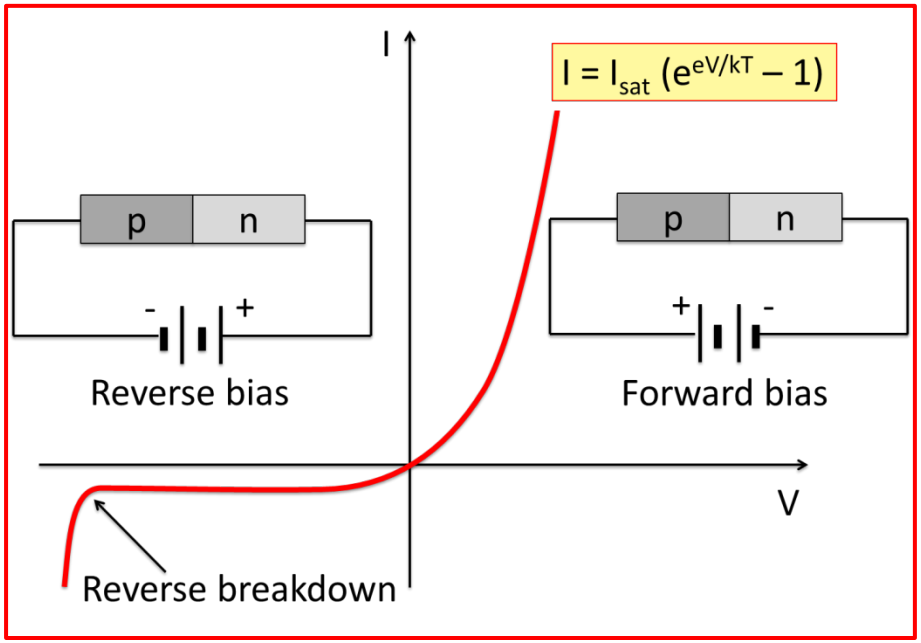
p-n junction – voltage biased

- Bias voltage (+ve on p-side) modifies effective junction potential $\phi_j^{eff} = \phi_j - V$
- This changes the recombination current (barrier height changes) but leaves generation current unchanged – minority carrier density on both sides remains the same
- Forward bias $J_{rec}^{(h)} = J_{gen}^{(h)} e^{eV/k_B T}$ recombination current outstrips generation current exponentially giving diode action (at zero bias $J_{rec}^{(h)} = J_{gen}^{(h)}$)
- Hence $J_{tot}^{(h)} = J_{rec}^{(h)} - J_{gen}^{(h)} = J_{gen}^{(h)} e^{eV/k_B T} - J_{gen}^{(h)} = J_{gen}^{(h)} (e^{eV/k_B T} - 1)$
- In reverse bias:

  $J_{rec}^{(h)} \to 0 \Rightarrow J_{tot}^{(h)} \to -J_{gen}^{(h)}$

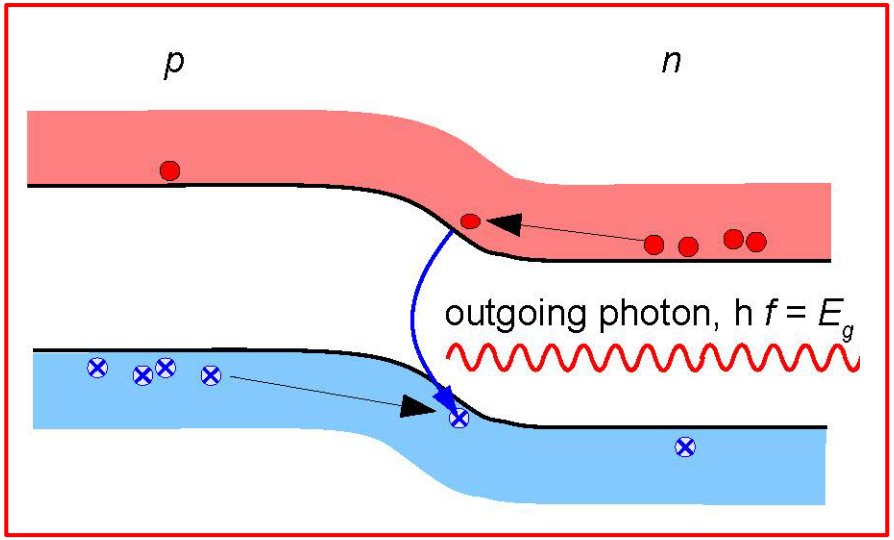
  current saturates at low level
- Diode equation – sum of hole and electron currents $I = I_{sat}\left(e^{eV/k_B T} - 1\right)$
- Saturation current

  $I_{sat} = J_{gen}^{(h)} + J_{gen}^{(e)} \propto n_i^2 \propto e^{-E_g/k_B T}$
- Reverse breakdown when reverse bias gets too large
- Mechanism - tunnelling of majority carriers across depletion zone



Light emitting diodes

- The inverse process to the photovoltaic effect powers light emitting diodes (LEDs)
- A current is injected into a p-n diode in a non-equilibrium situation where electron and hole chemical potentials differ by a large bias potential
- Electrons are injected from n-side to p-side of junction and holes in reverse direction
- Recombination of electron-hole pair occurs with emission of a photon with energy close to bandgap
- Not efficient for indirect bandgap semiconductor (Si, Ge)
- Direct bandgap III-V or II-VI materials used



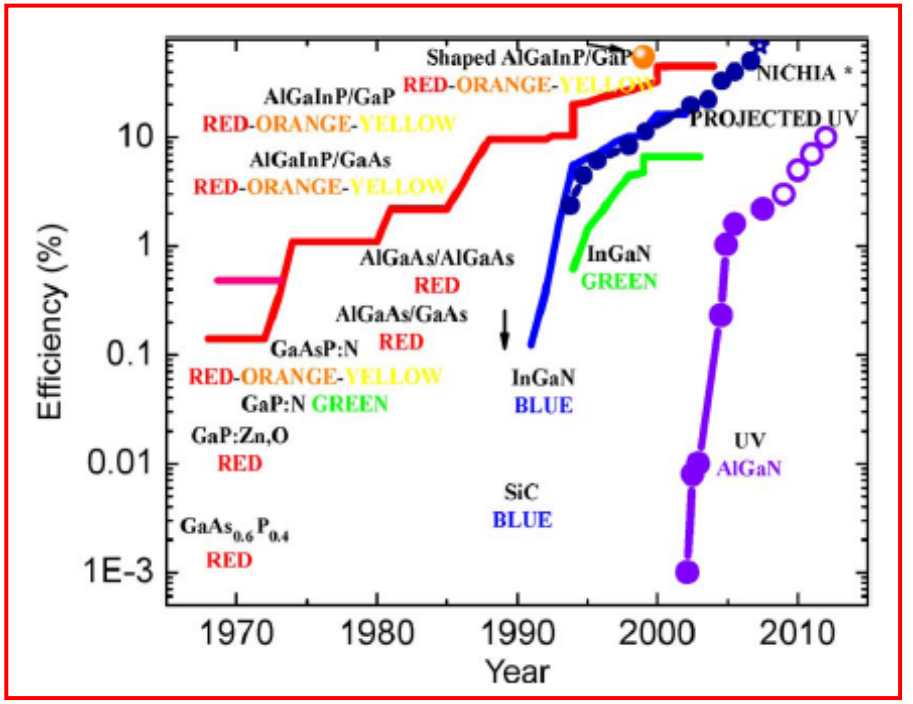
p-n junction in forward bias, electrons and holes injected into junction, photon emitted on recombination

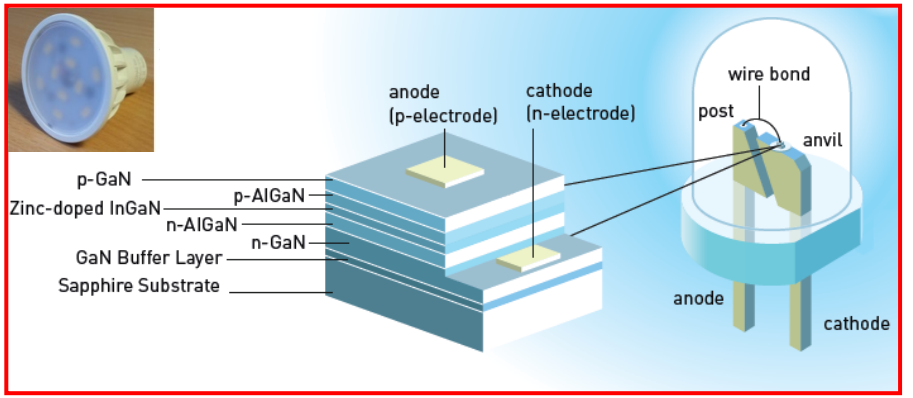
- Efficient LEDs have been developed across the visible spectrum - up to six times more efficient than incandescent bulbs
- Nobel prize in 2014 awarded for development of efficient blue GaN LEDs.

GaN LEDs

M S Slur et al IEEE trans Elect Dev. **57**, 12 (2010)

- Red and green LEDs developed since 1970s, used mainly as indicator lights
- Development of efficient blue GaN based LEDs over past 20 years has led to many applications in lighting
- High efficacy – 300lm/W (100lm/W in 2010) compared to 16 lm/W for incandescent lights and 70lm/W for fluorescent lights
- But for 'White' LEDs which combine blue light with yellow light from a phosphor only 80lm/W efficacy
- Lifetime upto 10x longer than Fluorescent lights
- *The future:*

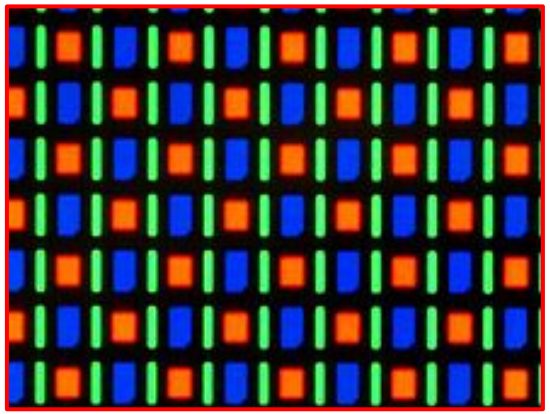


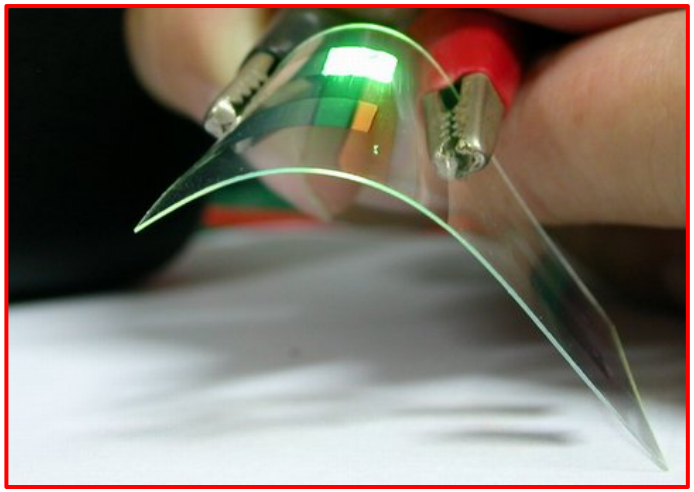


- Developing efficient green LEDs for red/green/blue spectral control
- Developing efficient UV LEDs for water purification and health applications

Organic LEDs

- Electroluminescent layer is a film of organic semiconductor
- Main applications in displays for TVs and mobile phones
- Works without back light - can be thinner and lighter than an LCD and achieve a greater contrast ratio and wider viewing angle
- Response times up to 1000x faster than LCDs
- Can be deposited on flexible substrates
- Based on either small molecules or polymers
- Problems with water damage and lifetime – gradual degradation
- Currently expensive but costs likely to fall with mass production





Summary of Lecture 15

- Metal to semiconductor contact
- p-n junction:
- Charge redistribution
- Band bending – equilibrium - balance of currents
- Effects of voltage bias
- Light emitting diodes: GaN, organic

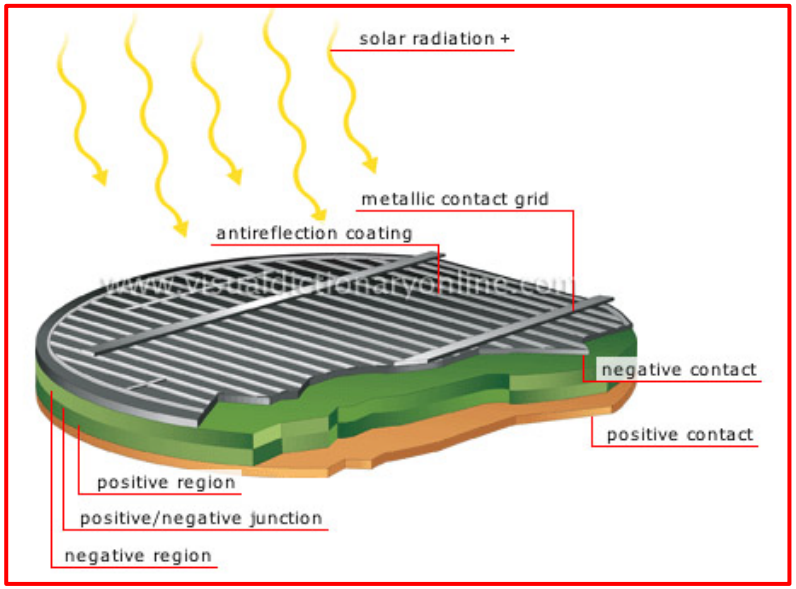
Quantum Condensed Matter Physics

Lecture 15

The End!

Quantum Condensed Matter Physics

Lecture 16



David Ritchie

Quantum Condensed Matter Physics

1. Classical and Semi-classical models for electrons in solids (3L)
2. Electrons and phonons in periodic solids (6L)
3. Experimental probes of band structure (4L)
4. Semiconductors and semiconductor devices (5L)

…. bending and equilibrium, balance of currents, voltage bias. Light emitting diodes; GaN, organic.

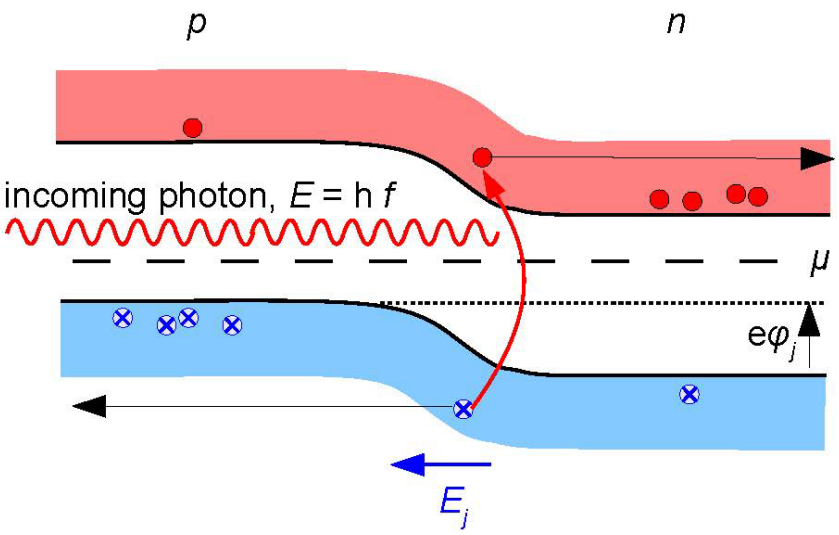
Photovoltaic solar cell; Shockley-Queisser limit, efficiencies, commercialisation. Field effect transistor; JFET, MOSFET. Microelectronics and the integrated circuit.

Band structure engineering; electron beam lithography, molecular beam epitaxy. Two-dimensional electron gas, Shubnikov-de Haas oscillations, quantum Hall effect, conductance quantisation in 1D.……

5. Electronic instabilities (2L)
6. Fermi Liquids (2L)

Photovoltaic solar cell

- Operation of a p-n junction based solar cell
- When illuminated each photon generates an electron-hole pair
- Pairs generated away from junction will recombine rapidly
- Pairs generated near junction separated by in-built electric field
- Electrons flow towards n-side, holes towards p-side
- This is equivalent to increasing the *generation* current, which flows in *reverse* (as opposed to *forward*) direction (n to p)

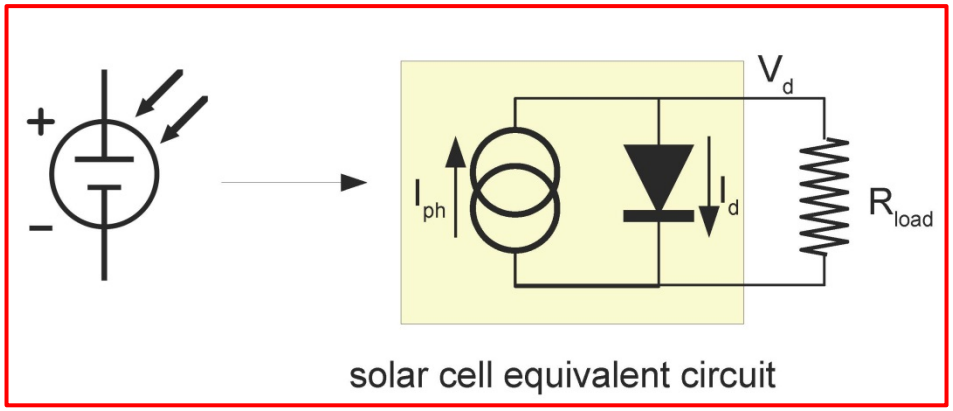


Electron-hole pair generated in junction region then separated by junction field E_j

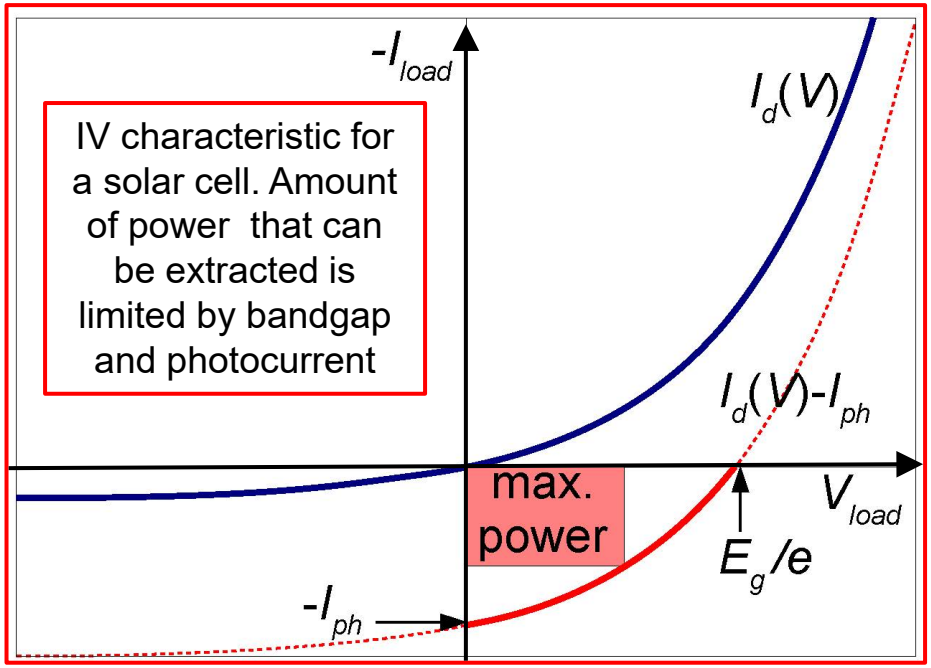
- Separation of charges across depletion region adds an extra dipole to system - like charging a capacitor and generating an overall electrical bias
- Induced voltage is in *forward* direction – because it is opposite in sign to the built in potential

Photovoltaic solar cell

- Model operation as a current source in parallel with a diode
- Current delivered I_{ph} depends on amount of light falling on junction area
- Consider IV characteristics
- For zero load resistance (short circuit, $I_{load} = I_{ph}$ but $V = 0$) and infinite load resistance ($I_{load} = 0$) no power is extracted
- What is open circuit V_d ?
- Upper limit given by band gap E_g - if V_d exceeds $\phi_j \sim E_g / e$ then the in-built junction field vanishes and photo generated carriers no longer swept out of junction area
- Maximum power extracted for ideally chosen load resistance is determined by quantity $I_{ph} E_g / e$

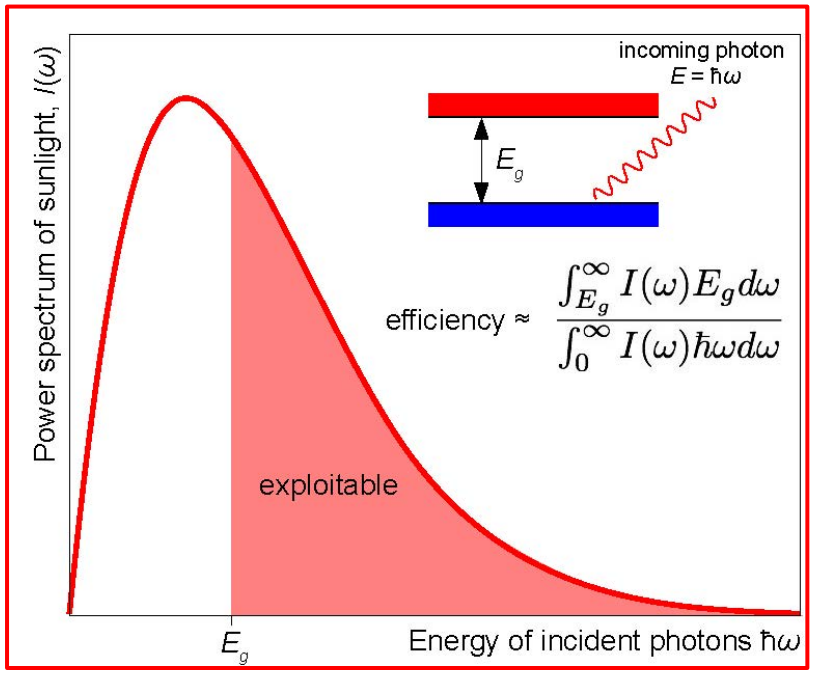


solar cell equivalent circuit



Solar cells: Shockley-Queisser limit

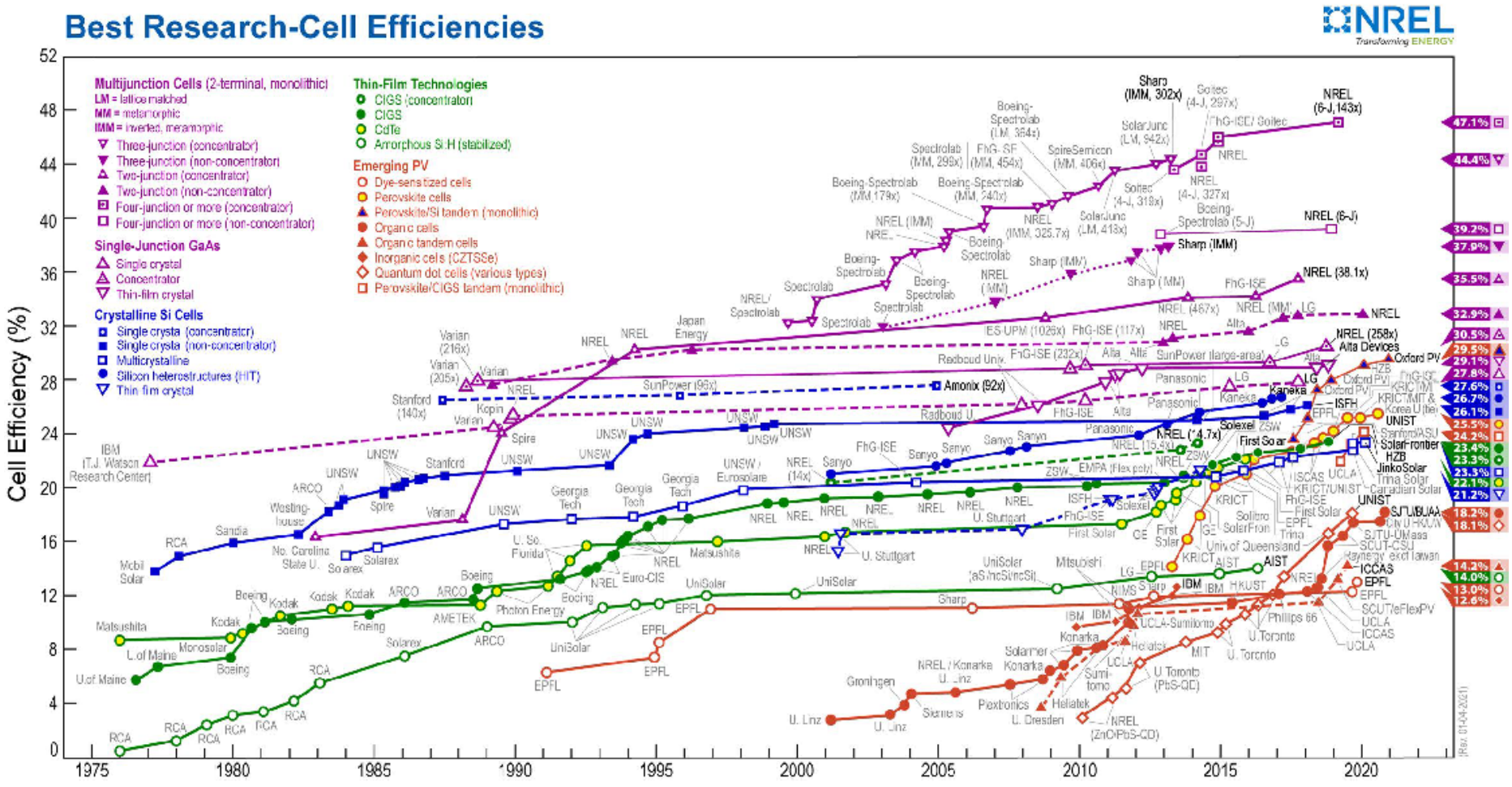
- How far is it possible to optimise solar cells by tuning the bandgap energy?
- Shockley and Queisser analysed the maximum efficiency of solar cells
- They considered matching between semiconductor band gap and intensity spectrum of sunlight
- Photons can only be captured if the bandgap is lower than the photon energy
- Power extracted depends on bandgap



- Also excited carriers in excited states well away from band edges will lose energy by decaying to lower lying states before leaving device
- Ratio of the energy extracted from sunlight $\int_{E_g}^{\infty} I(\omega) E_g \mathrm{d}\omega$ to the total energy incident on device $\int_0^{\infty} I(\omega)\hbar\omega \mathrm{d}\omega$ (where $I(\omega)$ is the spectral intensity) can be optimised as a function of E_g
- When combined with other limitations, optimum efficiency of single – junction solar cell is 33% for a bandgap of 1.2eV.
- Si solar cells achieve up to 22% efficiency

Shockley and Queisser, J Appl Phys, **510** (1961)

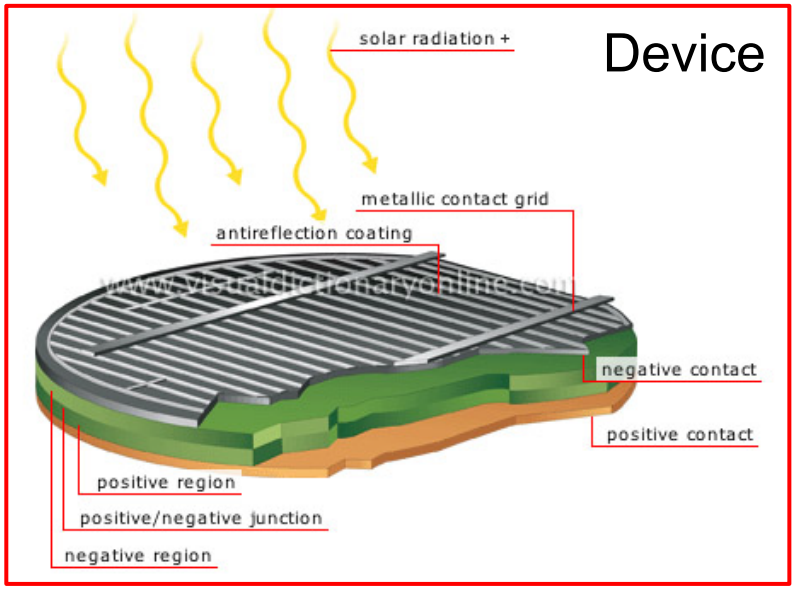
Solar cells- best research efficiencies

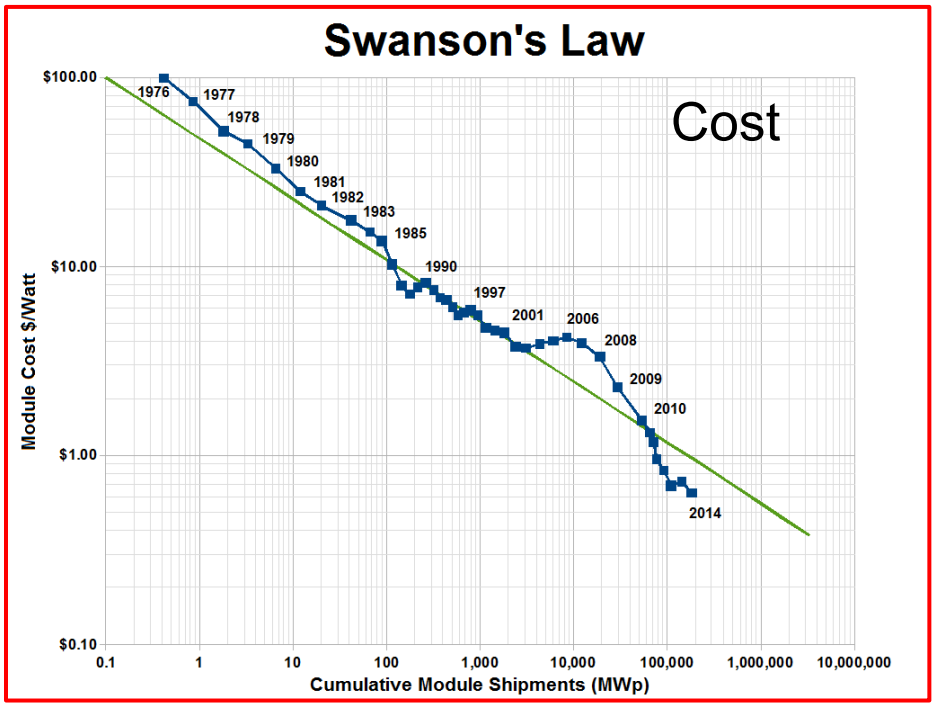


Conversion efficiencies of best research solar cells worldwide from 1976 through 2021 for various photovoltaic technologies. Efficiencies determined by certified agencies/laboratories

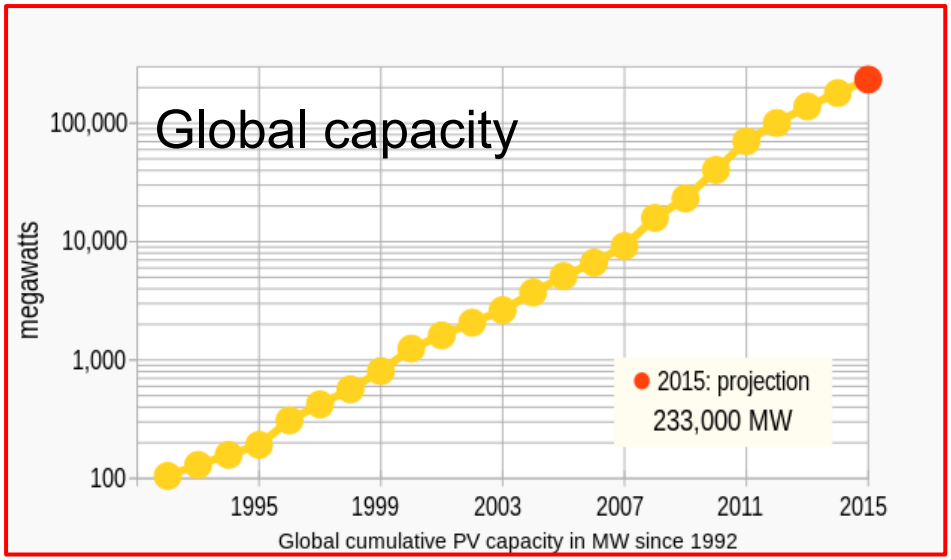
Sarah Kurtz and Keith Emery - National Renewable Energy Laboratory (NREL), Golden, CO, USA

Solar cells – commercialisation (1)





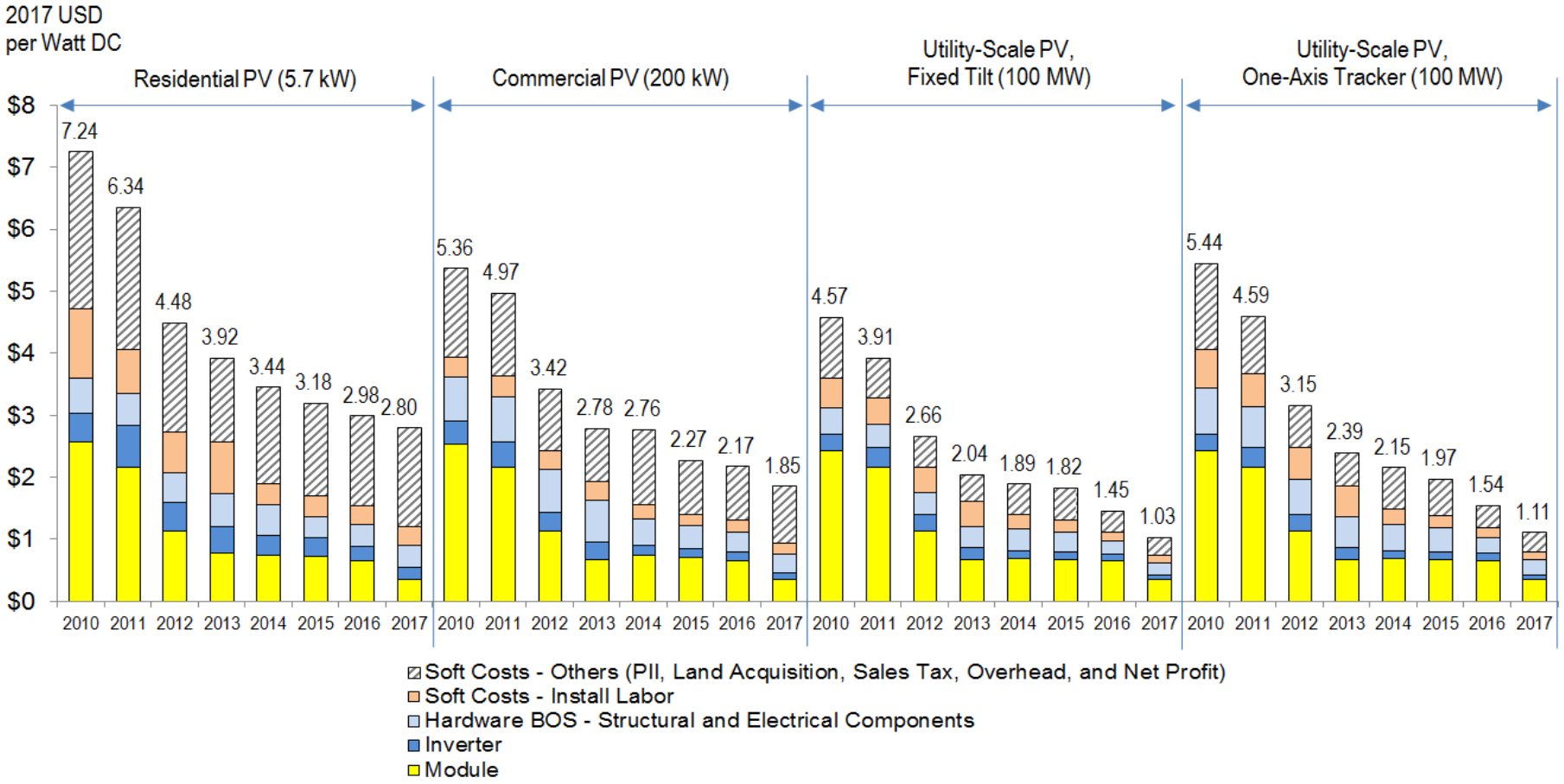
World's largest solar farm in Rosamond, California. 1.7 x10^6 solar panels over 13km^2. 579MW, power for 255,000 homes



wikipedia

Solar cells – commercialisation (2)

Reduction in Si cost reduces module cost to <$0.5/W



U.S. Solar Photovoltaic System Cost Benchmark: Q1 2017
Ran Fu et al.
National Renewable Energy Laboratory

Slide courtesy Dr Louise Hirst

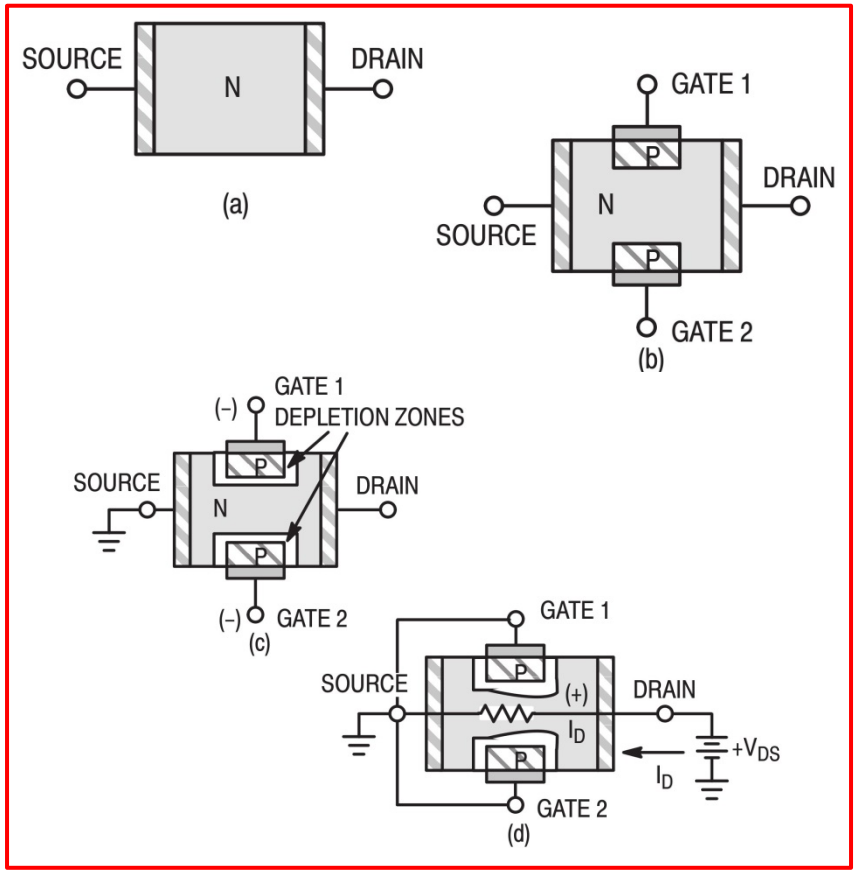
Field effect transistor

- Field effect transistors (FET) are the mainstay of the semiconductor industry
- Their principle of operation is based on our ability to manipulate the carrier density in a channel between two electrodes via a controlling voltage applied to a third electrode
- This controlling electrode is called the *gate*
- The electrode at which the mobile carriers (usually the electrons) originates is called the *source*
- The electrode towards which the carriers move is called the *drain*.
- A very readable account of the operation of FETs can be found at:

  http://www.freescale.com/files/rf_if/doc/app_note/AN211A.pdf.
- We distinguish between two types of FETs
- (a) junction based FETs (JFET), which use p-n junctions to control the width of the conducting channel
- (b) FETs in which the gate is separated from the rest of the device by an insulating layer, the metal-oxide semiconductor FET (MOSFET)

Junction field effect transistor (JFET)

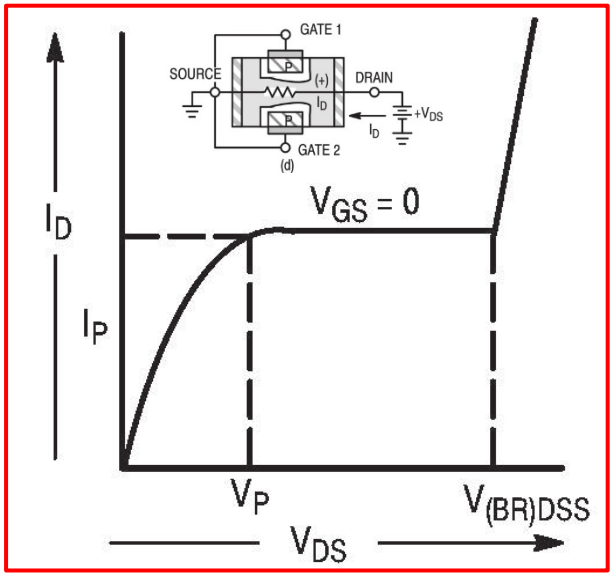
- JFET varies the current between source and drain by changing the width of the conducting channel
- Between source and drain in n-type semiconductor, electrical conductivity is high because of the high carrier density
- Adding p-type regions between source and drain contacts connected to gate electrodes, allows control over the current flow between source and drain
- At junction between p-type and n-type regions of the device, depletion zones form and conducting width of channel between source and drain is reduced

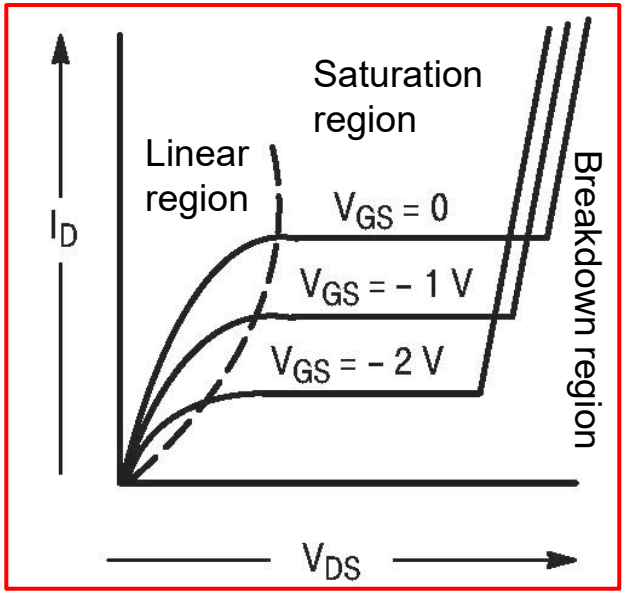


- By applying a voltage to the gate electrodes, depletion zone width can be controlled, altering the width of conducting channel:
- Positive gate voltage reduces the size of the depletion zone, and increases the current in the conducting channel
- Negative gate voltage widens the depletion zone and reduces the current

JFET operation

- Current voltage characteristics of an n-type JFET with p-type gate
- With increasing drain-source voltage V_{DS}, the drain-source current I_D rises roughly linearly, controlled by the gate-source voltage V_{GS}
- Increasing V_{DS} causes depletion regions to grow until they meet
- In this saturation region any further increase in V_{DS} is counterbalanced by an increase in the depletion region towards the drain
- The effective increase in channel resistance prevents any increase in I_D as V_{DS} increases
- V_{DS} that causes the current limiting condition is known as the *pinch-off voltage* V_P
- If V_{DS} is too high breakdown region is entered and I_D increases rapidly
- I_D controlled by V_{GS} in saturation region is operational mode for JFET in amplifier circuit



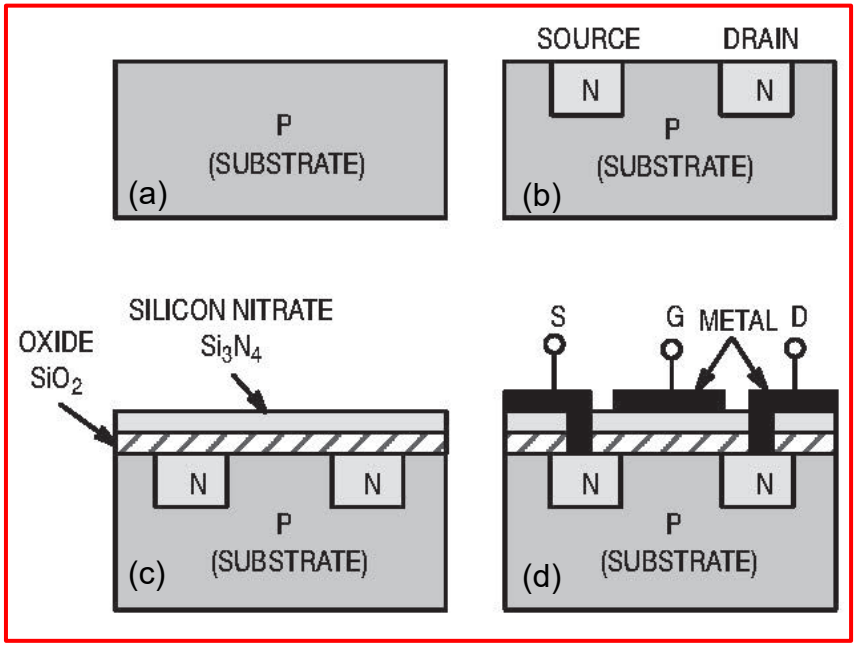


Metal oxide semiconductor field effect transistor (MOSFET)

- MOSFET – very commonly used in modern electronics
- Width of the conducting channel between source and drain controlled by electric fields using a gate electrode insulated from the rest of the device
- No current flow from the gate electrode - extremely high input impedance
- In contrast in the JFET - where a small current flows across depletion region

Manufacturing process for enhancement mode n- channel MOSFET

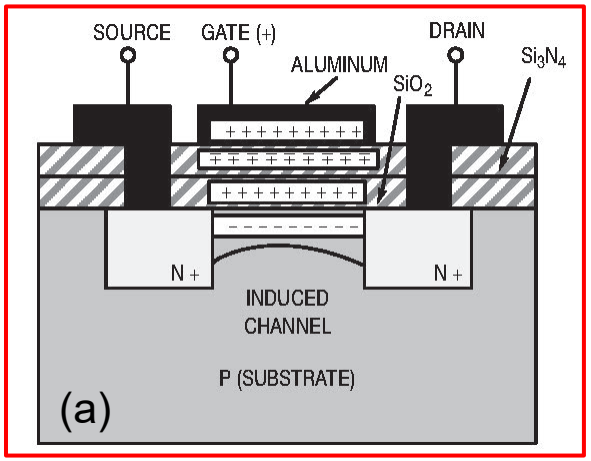
- (a) p-doped substrate (b) n-doped source and drain contacts by ion implantation
- (c) insulating silicon oxide layer deposited followed by insulating silicon nitride – stops sodium diffusion
- (d) metallic contact made to source and drain through holes in insulator
- Gate electrode insulated from substrate

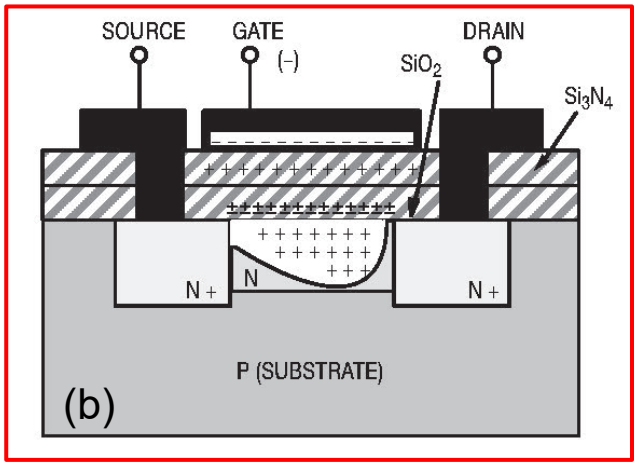


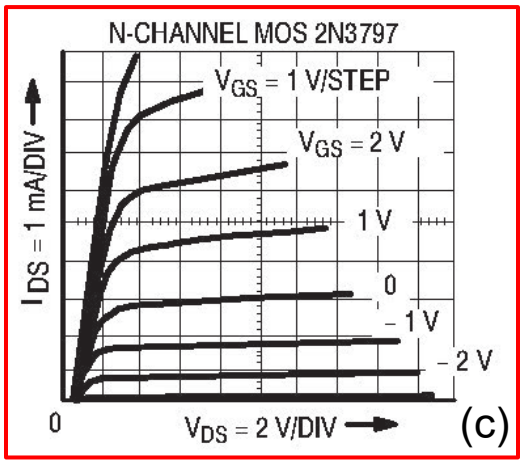
- Applying a +ve voltage to the gate pulls electrons into the depleted zone and establishes a conducting channel between source and drain

MOSFET operation

- Several MOSFET designs used, operation relies on two principles:
- (1) By changing gate voltage, depleted regions between source and drain electrodes can be filled with carriers or depleted of carriers. Allows variation of the resistance of the source-drain channel
- (2) As for the JFET, pinch-off occurs near the drain electrode, causing the source-drain current to saturate making the device useful as an amplifier



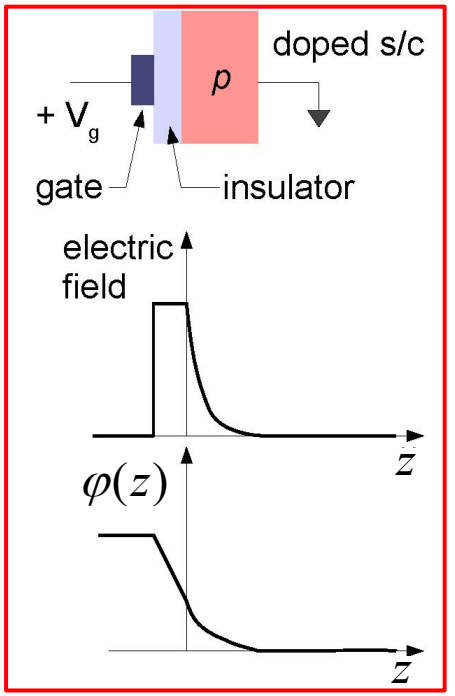


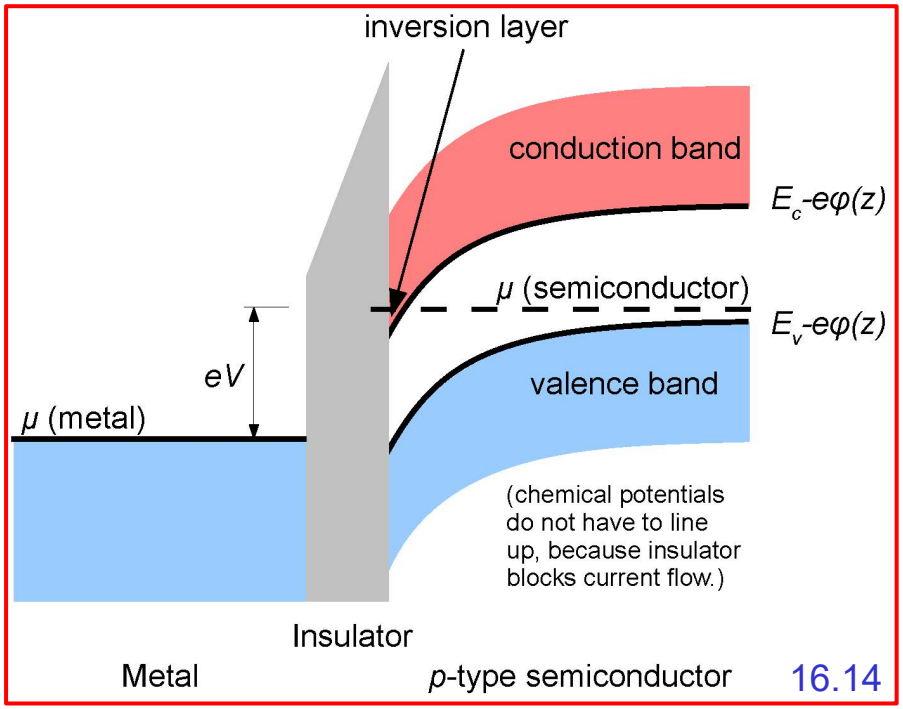


- (a) enhancement mode MOSFET, +ve voltage pulls minority carriers towards surface forming high conductivity *inversion layer* channel
- (b) depletion – enhancement mode MOSFET: -ve voltage depletes channel, increasing resistance, +ve voltage enhances channel, reduces resistance
- (c) Typical IV characteristic for depletion-enhancement mode MOSFET, note pinch-off at high source – drain voltage

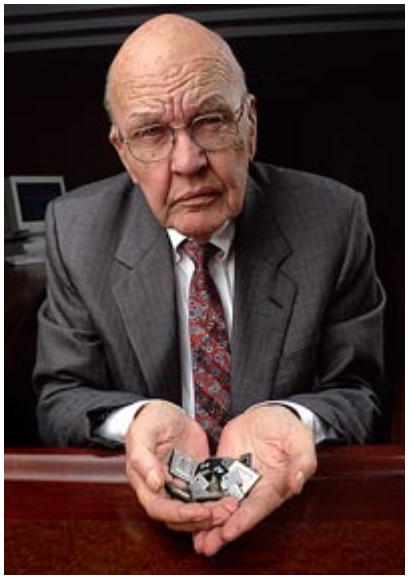
MOSFET- inversion layer

- Band bending in a MOSFET
- Applying a +ve voltage to the gate electrode creates an electric field across the insulating oxide layer
- This field penetrates some distance into the semiconductor
- This field sets up a varying potential $\varphi(z)$ close to the surface of the semiconductor
- If the resulting band-bending at the semiconductor/oxide interface becomes larger than the band gap E_g the conduction band edge falls below the chemical potential at the surface causing an *inversion layer* to form
- Width of inversion layer can be controlled by gate voltage but is narrow enough so quantisation effects are observed

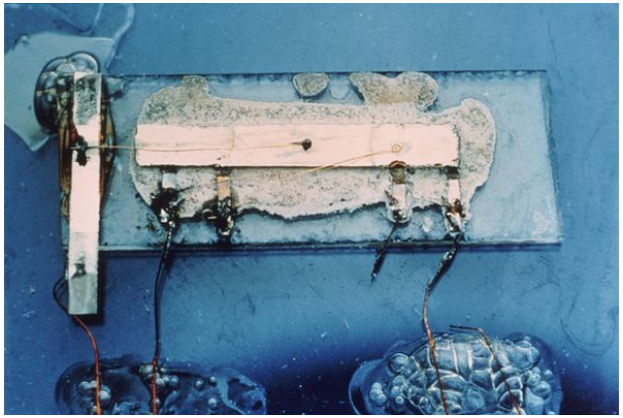




Microelectronics – the start

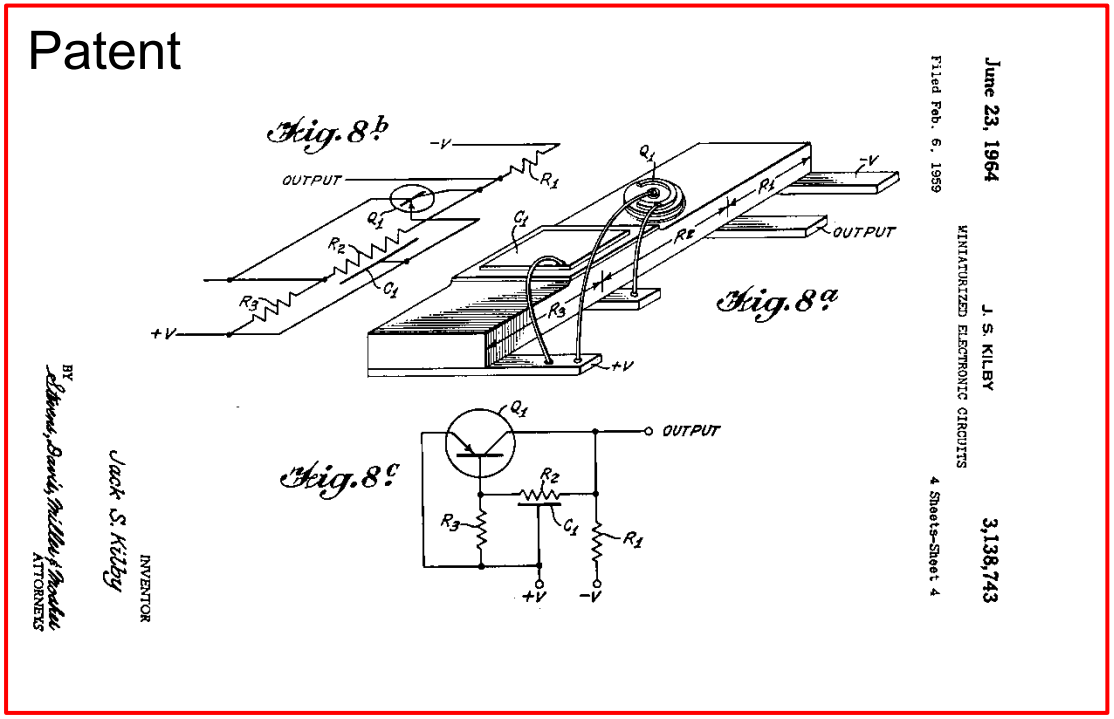


- First integrated circuit developed by Jack Kilby at Texas Instruments in the summer of 1958 (he wasn't allowed a summer holiday….)
- Developed combination of a transistor, resistors and capacitor on a single piece of germanium which produced a sinewave voltage – an oscillator
- Also inventor of handheld calculator and thermal printer…….



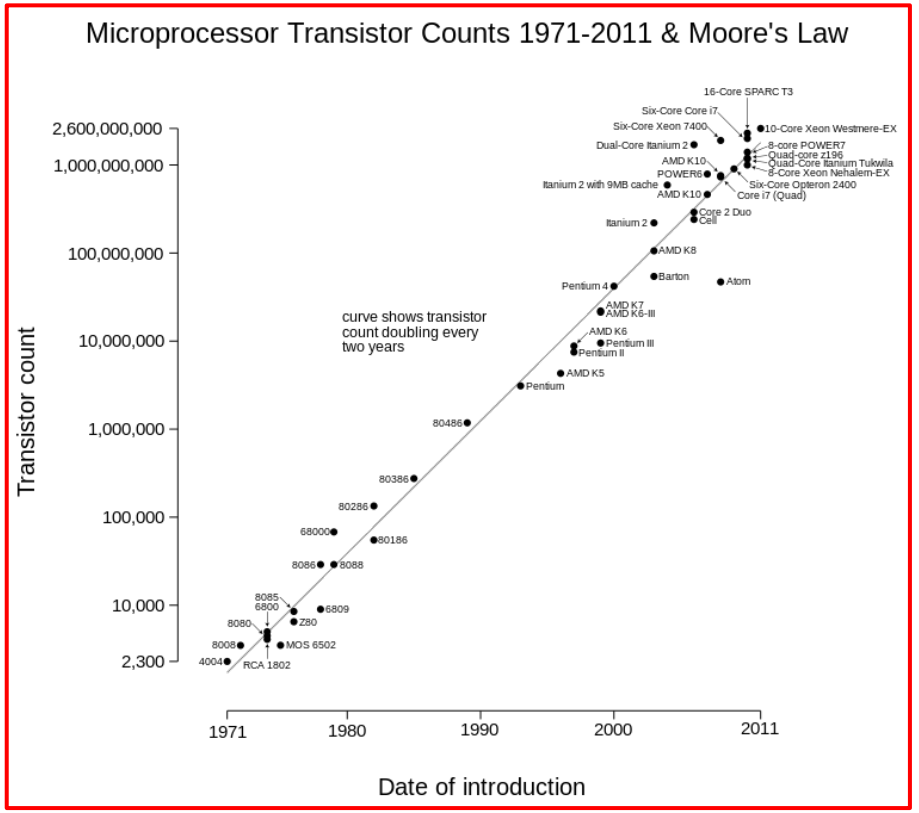
First integrated circuit – an electrical oscillator, Jack Kilby Texas InstrumentsI1958 (Nobel prize 2000)

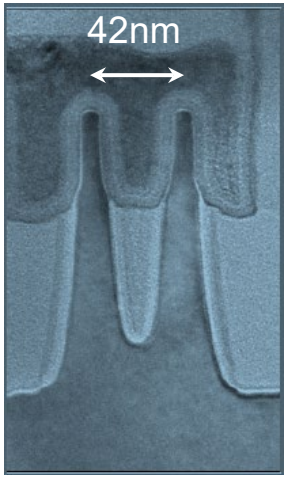
Patent



Microelectronics – now

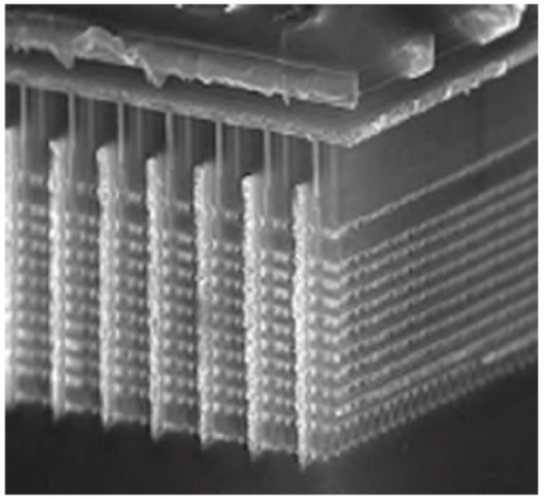
- No. of transistors in an IC risen by a factor of 10^6 in 40 years
- Development being slowed by difficulty of making small enough features – currently 10nm process being developed, 7nm has been demonstrated, 5nm being thought about….
- Will run into quantum tunnelling problems at 7nm – may be the end of Moore's Law…..
- 3D devices and architectures being developed to increase density
- Materials with higher mobility than Si required to increase speed and reduce power dissipation – Ge or InGaAs





14nm trigate transistor

Samsung's 32-Layer 3D V-NAND Memory Chip



Source: PC Perspective, Samsung

Summary of Lecture 16

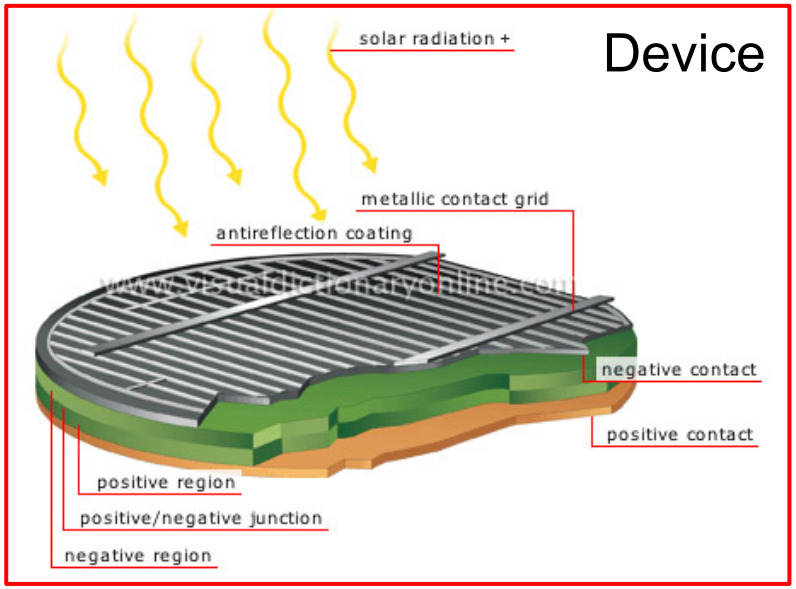
- Photovoltaic solar cell - Shockley-Queisser limit, efficiencies, commercialisation
- Field effect transistor - JFET, MOSFET
- Microelectronics and the integrated circuit

Next term – Final 6 lectures

- Semiconductor devices and low-dimensional physics
- Electronic instabilities - charge density waves, magnetism
- Fermi liquids - collective excitations and heavy Fermion materials

Quantum Condensed Matter Physics

Lecture 16



Have a good vacation!